2016

19th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 7057

Problem Chosen: B

Summary

Our team was tasked to minimize the number of warehouses needed to offer one-day ground shipping throughout the contiguous United States for an expanding recreation company. We created a method to determine the location of the warehouses based on its ZIP code and the location's proximity to a major roadway. Given the ranges of ZIP codes in each state, we found the most central ZIP codes in the given range and examined cities within that range whose proximity was close to major roadways. After implementing this method, we found that the company would need to build 28 new warehouses. However, three of those warehouses would exclusively cover uninhabited area, and therefore would be a waste of money and materials. This reasoning decreased the number of warehouses to 25 new warehouses, roughly the same as the amount of FedEx ground hubs [2].

After establishing the locations of the warehouses, we analyzed the tax rates and populations of each state. A metric, L, was created for each state to rank the effect of tax liability on the total consumer base. By calculating the metric for every state, we found which states had the most people and the highest sales tax. To improve consumer sales, warehouses were moved to decrease tax liability for certain states that had high per capita and total tax liability. The metric was modified once we took into account clothing and the apparel tax for every state.

Our final solution places warehouses so that the company can offer one-day ground shipping to the entire inhabited continental United States of America. We determined the locations based on a method that combines physical connectivity with relative placement in the state. Taking into account the sales and apparel taxes of each state, we optimized the locations of our warehouses while decreasing total consumer tax liability.

Dear M. President,

Thank you for allowing us to aid you in the decision of where to build the new warehouses. As you know, the successful placement of new warehouses is vital to the continued growth of your company. By providing one-day shipping to all four corners of the nation, we will be able to drastically increase our customer base.

After carefully analyzing this problem, we have determined that the best solution is to build an additional 25 warehouses in select cities across the nation. These 25 warehouses will take into account the varying state taxes for both sales tax and apparel tax exemptions, striving to provide the most number of customers with the cheapest option for your items so that they are more inclined to buy again from your company.

This solution allows the entire nation to be served with only 25 additional warehouses (Fig. 9). This covers all *inhabited* land in the nation, as opposed to all land in the nation. There are several places in the nation, most notably places in Idaho, Nevada, and Maine, that are difficult to reach, mainly due to mountainous or forested terrain. Because of this, the only realistic way to ensure one-day shipping to these locations is to build a warehouse exclusive to these parts of each state. However, these three locations correspond with a significant decrease in population, to the point that it is unadvisable to put forth the time and money required to build another warehouse.

While this solution attempts to minimize the number of warehouses needed, it still requires that 25 additional warehouses be built, which is impractical to do all at once. While these could be built east to west, gradually pushing westward, there is a better way to disperse the warehouses. By building only four of these additional warehouses at Denver, Colorado; Marion, Arkansas; Fresno, California; and Janesville, Wisconsin, we are able to provide two-day shipping to a vast majority of the contiguous United States. Doing so will lead to an increase in our customer base and profit that we could use to build the other 21 warehouses and provide one-day shipping nationwide.

The adoption of our recommendations would allow for your company to move into its next stage as safely as possible, minimizing the risk as your net profit increases. We hope that you consider our recommendations as you guide your company through this exciting period.

Thank you,
Team #7057

Introduction and Restatement of Problem

Online commerce has increased greatly with the growth of the Internet. It offers the ability to purchase items without going to a store, saving the consumer time. However, online shopping brings into account other factors, such as the shipping location and the tax rate for the items. A warehouse in Oregon will have different implications than a warehouse in Ohio for a shopper in Ohio. A recreation equipment company is looking to expand its online business by building warehouses across the United States. The company wishes to offer one-day ground shipping to the 48 states of the continental U.S. The goal of our paper was to find the optimal number and locations of warehouses for a recreation equipment company that would cover the entirety of the contiguous United States, using the UPS Ground Time-in-Transit maps [1].

Assumptions and Justifications

In the analysis of this problem, certain assumptions were made that reduced the number of variables taken into account.

Assumption 1:	The approximate delivery areas for one-day shipping are accurate on the given UPS website [1]. That is, if an item is claimed to arrive in one day it will always arrive within one day.
Justification:	We must assume that the times provided to us by the official website are accurate in order to have a source of data.
Assumption 2:	Variances in speed limit, traffic flow and routes are negligible when calculating delivery date and any outside occurrences (hurricanes, accidents, detours, etc.) that will drastically affect delivery date will not occur.
Justification:	It is impossible to predict how these factors will affect our model on any given day as they change frequently. By removing them, we can simplify our model. Furthermore, it is likely that the estimated delivery date received from the UPS website already takes these factors into account. While such factors do naturally occur, they are impossible to predict and generally unlikely to occur.
Assumption 3:	The warehouse in New Hampshire cannot be moved.
Justification:	The company's headquarters are in New Hampshire, and moving this would be an unnecessary expense for the company.
Assumption 4:	There is no limit to the number of people that a warehouse is able to service. In other words, if a warehouse is able to ship to an area

it will always have the resources available to do so no matter how much demand they receive.

Justification: This is a factor which affects the delivery date. However, it is based upon the current demand for the product which fluctuates and is thus hard to incorporate into our calculations, especially without prior knowledge of demand for these products. Removing this factor allows us to simplify the calculations. Furthermore, it is likely that the company will research the surrounding area before building a warehouse and will adjust the size of the warehouse accordingly.

Assumption 5: The number of consumers in each state is proportional to the state's total population.

Justification: Although different states may be more inclined to buy the company's recreational equipment, for the sake of the metric and measuring taxes we assume that the state's population correlates with the number of consumers from that state to simplify calculations.

Assumption 6: The cost of building a warehouse is equal in all areas of the United States.

Justification: In reality, the cost of real-estate would be much higher in certain areas. We will disregard the cost of property in order to obtain the best and/or largest one-day shipping area.

Assumption 7: The shipping occurs on Monday, November 14, 2016, which is used on the UPS website to find one-day shipping area.

Justification: We wish to model the situation that occurs most of the time, which is not during "rush season," or between Black Friday and Christmas. Shipping also only occurs during business days, so sending a package on Friday would be received by Monday. We chose November 14, 2016, as it is not during "rush season" and is a Monday, so it is uninterrupted by non-business days.

Assumption 8: Each state buys the same fraction of apparel versus outdoor gear.

Justification: Different states could buy more or less gear than other states, but for the purposes of this project we are equalizing the percent of apparel bought by each state.

Placement of Warehouses

Our goal was to place the minimum number of warehouses that could offer one-day ground shipping throughout the contiguous United States, and when met, will indicate the success of this method. Cities with large populations are more likely to be connected with other areas as people frequently travel to and from such cities, so we first placed warehouses in the twenty largest cities of the US. However, the cities tended to be close to one another, and the areas for one-day ground shipping consistently overlapped (Fig. 1).

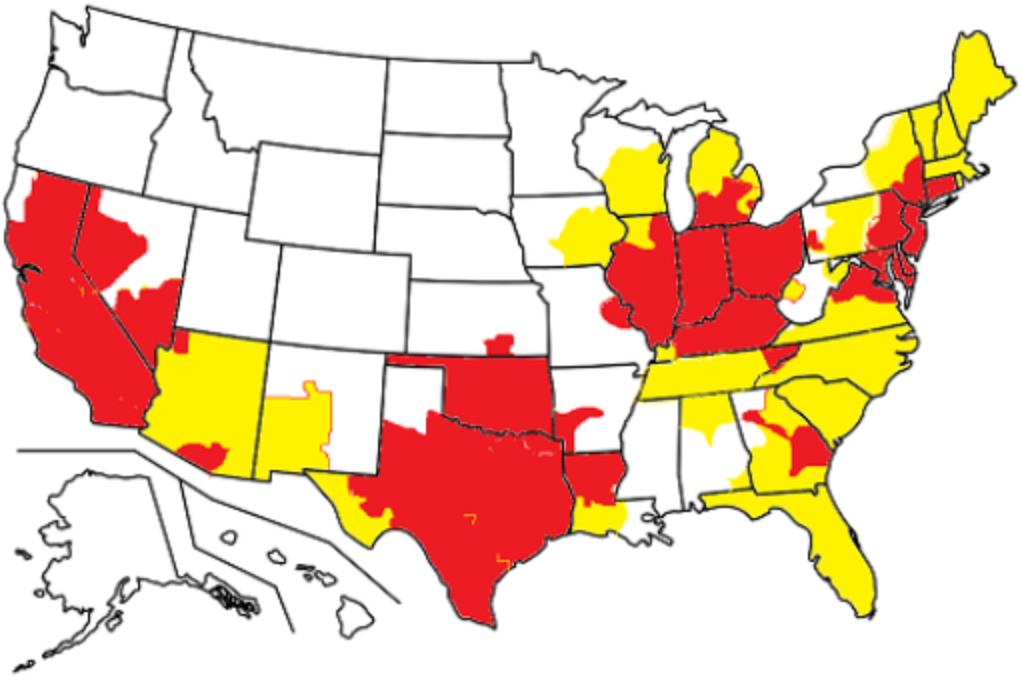

Figure 1: One-day shipping zones (yellow) when warehouses are located at 20 most populated cities. Locations where two or more zones overlap are shown in red.

Afterwards, we chose to add warehouses on cities close to major roadways and interstates. We hypothesized that places near the intersection of highways would be easier to access than warehouses away from major roadways (Fig. 2).

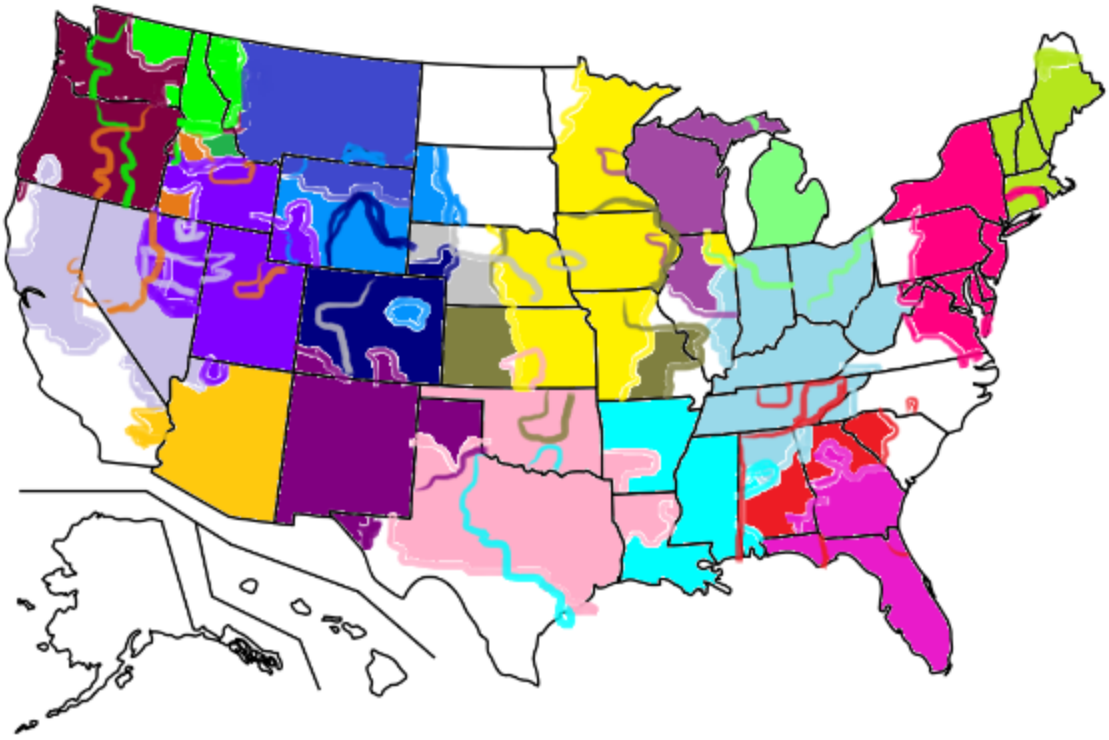

Figure 2. One-day shipping zones based on cities near major highways and interstates, each color is the reach of one warehouse. Colored lines in one area are the overlaps of each zone.

This method worked well for some areas, like the light blue region near Indiana, Ohio, West Virginia, Kentucky, and Tennessee (warehouse location: Lexington, KY). However, this made it harder to cover small sections of the US, and wasted warehouses on unnecessary locations, like the small dark green section in Idaho (warehouse location: Idaho Springs). This gave us an idea for our final and most successful method.

When we based our warehouse locations on their proximity to major highways and interstates, there was a lot of coverage that was provided. However, we reached a point where we had predicted the addition of over 30 warehouses and we still had more areas to cover. We felt that while it did provide a good amount of coverage in some areas, we could add another component to the method that would make it more optimal and selective. Because the ZIP code of a warehouse determines how far surrounding cities can be from the warehouse in order to qualify for one-day shipping, we decided to combine ZIP codes and highways in order to better optimize our outcomes.

ZIP codes are numbers created by the US Postal Service in order to facilitate the delivery of mail, increasing as one moves east to west in the United States. For this method, we examined the first three digits. The first digit represents its national area and the second and third digits

represent the sectional center or large city post office that the mail will be sorted at. Using the ZIP code system, we worked our way from west to east, starting with Utah. Utah was selected because it is fairly central in the west half of the country, which has fewer major highways and roads than the East side and is thus more difficult to ship to. Our final method consisted of finding the central ZIP code in each state according to the map of ZIP codes provided (Fig. 3).

Figure 3: Map of ZIP code ranges for all states.

After finding the mean ZIP code, we allowed a ± 20% range (Table 1), which allowed for freedom in states that are oddly shaped, have inconsistent labelling patterns or have few major roadways, and used the ZIP code within that range that was closest to an interstate or highway. The closer a ZIP code is to the median amount, the more preferable it was. It was most optimal to find a location near an intersection of highways and interstates. We tested all possible ZIP codes by putting them into the UPS website [1] and compared them to determine which covered the most area.

State	Original Start	Original End	Middle	20% range	Modified Start Range	Modified End Range
Florida	32	34	33.0	0.4	32.6	33.4
Colorado	800	819	809.5	3.8	805.7	813.3
New Mexico	870	884	877.0	2.8	874.2	879.8
Montana	590	599	594.5	1.8	592.7	596.3
N Dakota	580	589	584.5	1.8	582.7	586.3
S Dakota	570	579	574.5	1.8	572.7	576.3
Wisconsin	530	549	539.5	3.8	535.7	543.3
Iowa	500	529	514.5	5.8	508.7	520.3
Kentucky	400	429	414.5	5.8	408.7	420.3
Tenessee	370	385	377.5	3.0	374.5	380.5
Texas	733	885	809.0	30.4	778.6	839.4
Michagain	480	499	489.5	3.8	485.7	493.3
New Hampshire	30	38	34.0	1.6	32.4	35.6
Maine	39	49	44.0	2.0	42.0	46.0
Pennsylvania	150	196	173.0	9.2	163.8	182.2
West Virginia	246	269	257.5	4.6	252.9	262.1
California	900	961	930.5	12.2	918.3	942.7
NC	270	289	279.5	3.8	275.7	283.3
Oklahoma	730	749	739.5	3.8	735.7	743.3
Arizona	850	869	859.5	3.8	855.7	863.3
Washington	980	994	987.0	2.8	984.2	989.8
Idaho	832	839	835.5	1.4	834.1	836.9
Indiana	460	479	469.5	3.8	465.7	473.3
Illinois	600	629	614.5	5.8	608.7	620.3
Alabama	350	369	359.5	3.8	355.7	363.3
Kansas	660	679	669.5	3.8	665.7	673.3
Nebraska	680	699	689.5	3.8	685.7	693.3

Table 1: List of original vs. modified ZIP code ranges based on the ± 20% range allowed per state.

We followed this method for all 48 states except for the corner and irregularly shaped states of Maine and Florida. Below, our final map (Fig. 4) shows the areas covered by the ZIP codes listed in Table 2.

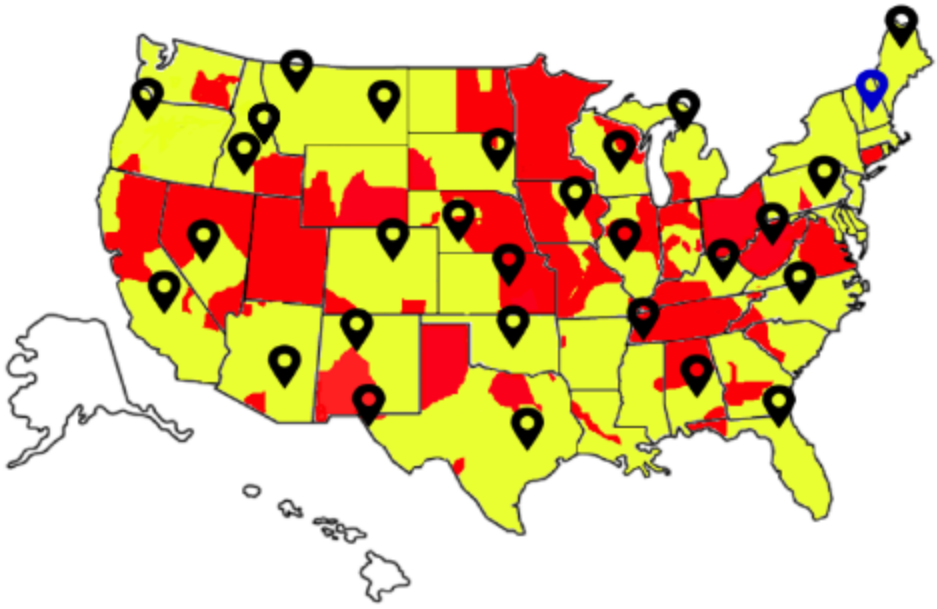

Figure 4: Map of contiguous United States covered by the addition of 28 warehouses. The areas shaded yellow illustrate the areas covered by one-day shipping, and the areas shaded red illustrate the areas that are covered by more than one warehouse. The blue marker represents the original warehouse in New Hampshire

State	Zip Code	City	State	ZIP Code	City
Florida	32068	Jacksonville	*Maine	04736	Caribou
Colorado	80204	Denver	Pennsylvania	18062	Macungie
New Mexico	87107	Albuquerque	West Virginia	26104	Parkersburg
Montana	59832	Drummond	California	93722	Fresno
Montana	59330	Glendive	North Carolina	27518	Cary
Texas	77002	Houston	Oklahoma	73119	Oklahoma City
Texas	79901	El Paso	Idaho	83301	Twin Falls
*Nevada	89406	Fallon	Arizona	85004	Phoenix
Kentucky	40503	Lexington	Oregon	97214	Portland
Iowa	50309	Des Moines	*Idaho	83469	Shoup

South Dakota	57105	Sioux Falls	Illinois	62049	Hillsboro
Tennessee	38104	Memphis	Alabama	36117	Montgomery
New Hampshire	03301	Concord	Kansas	66606	Topeka
Michigan	49781	Saint Ignace	Nebraska	69101	North Platte
Wisconsin	53548	Janesville			

Table 2: A list all the ZIP codes and the corresponding cities that allow for one day shipping to the entire contiguous US using our method. Cities with asterisks are uninhabited.

Table 2 shows the list of all locations of our proposed warehouses. States with asterisks next to them indicate that the warehouse could be disregarded. After finding the warehouses that covered the entirety of the contiguous US, we thought of ways to minimize the addition of 28 new warehouses, one of which included looking at areas of the US that were uninhabited. After overlaying our final solution on a map that had all the uninhabited areas, we found that the warehouses in Nevada, Maine, and Shoup, Idaho were in areas that, for the most part, have a population near zero, as seen below (Fig. 5).

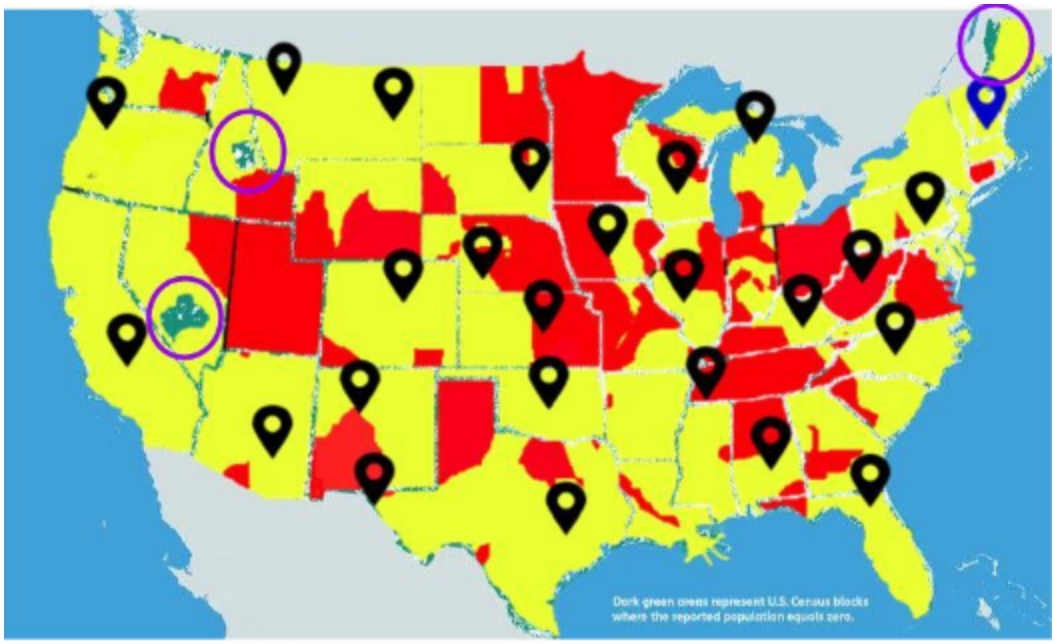

Figure 5: Map of all covered territory except for part of Nevada, Idaho, and Maine (colored in green) due to the fact that they are, for the most part, uninhabited.

Having a total of 26 warehouses is an acceptable amount due to the fact that in order for one day shipping to be available for all inhabited parts of the United States, the warehouses must be close

to a hub. A hub is a facility that processes packages for a shipping company. When a package is shipped, it is first sent to the hub and then delivered to the final location as opposed to making a trip directly to the location. A similar service to UPS, FedEx has 25 hubs across the nation as seen in figure 6 [2]. It stands to reason that UPS would have a similar number of hubs placed across the United States, and it follows that we should have a similar number of warehouses - approximately one per hub.

Figure 6: Map of all ground hubs for FedEx.

Justification of Model One

The combination of ZIP codes and their proximity to interstates and highways was done in order to improve the interstate/highway method by making it more selective. The interstate/highway method was pretty effective in that it had great coverage. However, there were some areas that couldn't be covered well or didn't have many intersections near them. We used ZIP codes because the built-in feature on the website of the United Parcel Service calculates the areas that receive one-day shipping based off of the ZIP code of the warehouse building. In order to efficiently combine ZIP codes and the interstate/highway method, we wanted to use the central ZIP code within each given range per state. We couldn't use the middle point of each state due to the fact that each state is structured and shaped differently. Therefore, we had to put a rule into place that would standardize all the states to some degree. In order to accomplish this, we decided to take the middle range of the ZIP code range per state [5].

We also decided to take into account uninhabited areas due to the fact that there were small areas in states such as Idaho and Nevada that wouldn't be reached until a day later when compared to the areas around them. After looking into these cases, we found that there were geographic

barriers such as mountains and forests that separated that part of the state from the rest and that they required their own warehouses in order to be covered for one-day shipping. While there were a lot of uninhabited areas in the west, a majority of them were covered by warehouses in other areas. There were three uninhabited areas that required their own warehouses due to their isolation. When taking this into account, it doesn't make sense for a company to build an entire warehouse in an area that is not populated.

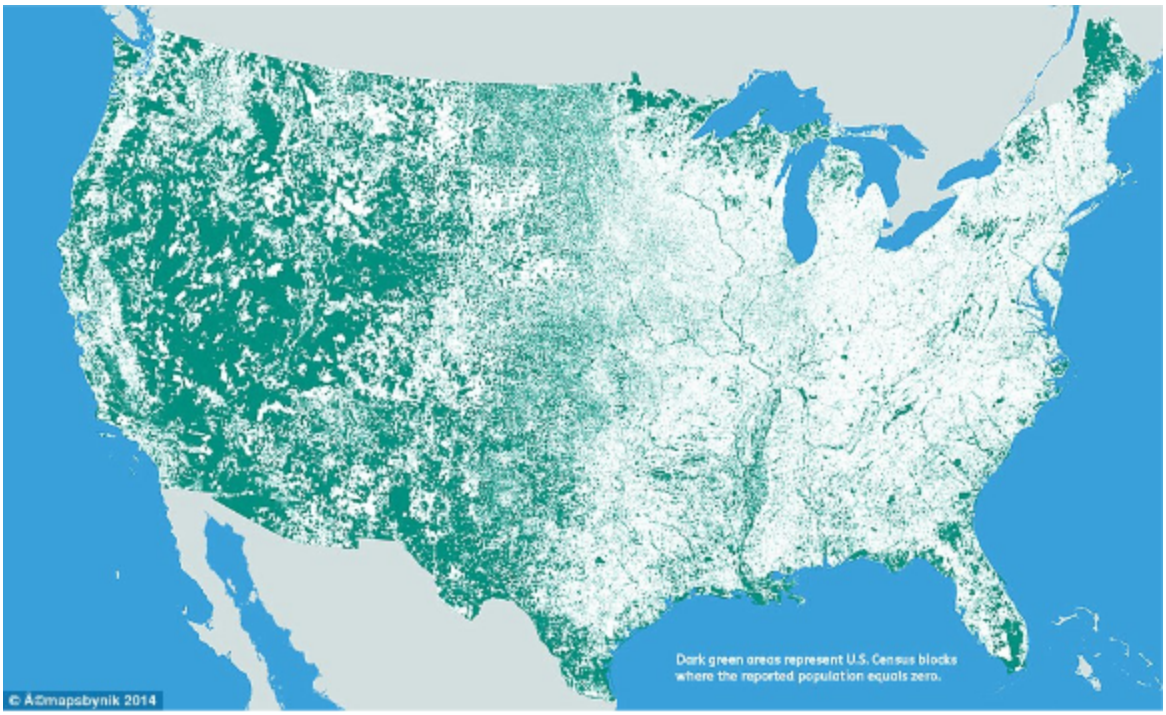

Figure 7: Map of the unpopulated areas (green) of the United States based on the 2010 US Census.

Tax Liability

The tax liability is the amount a consumer must pay to a tax authority as a result of a taxable event [7]. The company will attract more consumers in a certain state if we can remove the warehouse in state, because the sales tax for that company would become zero. In this section, we attempt to find which states would most benefit from not having to pay taxes for the company's products. Since we assume that each state's population is proportional to the number of consumers in that state, we can create a metric L that describes the total tax liability of consumers in that state,

$$L = \left(\frac{p_{state}}{p_{total}}\right) * r$$

(Eq. 1: The tax liability of consumers in a state based off the total consumer base)

where p_{state} is the population of the state, p_{total} is the total population of the country, and r is the tax rate multiplied by 100 (so for a tax rate of 4%, r is equal to 4). The state populations were

based off of the 2010 Census [8]. We assume that the consumer base for a state and the tax rate of a state have equal weights on the metric L, which means that a state with 5 consumers and a tax rate of 6% will have the same tax weight as a state with 10 consumers and a tax rate of 3% for the company.

We then calculated L for each state (See Appendix 1). A high L value means that a state has a higher total tax liability, based on its fraction of the country's population and its tax rate. Some states, such as North Carolina or New York, have high L values because they have large populations, which means the gross amount of taxation for those states is very large because more people are taxed. Other states, like New Jersey or Michigan, have lower populations than most states with high L values, but have higher tax rates making the per consumer taxation greater. Other states, like California and Florida, have high sales tax rates and very high populations.

With our current placement of warehouses (Fig. 5), the average sales tax that a consumer must pay anywhere in the United States is 3.76%. This average was calculated by finding the L value of every state with a warehouse, and then summing those numbers. Since consumers in a state with no warehouse have to pay no tax, their weighted taxes are zero. The sum is the average sales tax any consumer in the US must pay. However, the actual sales tax a consumer pays is completely based on their location. For example, the 12.3% of the US population that lives in California must pay a sales tax of 7.5%.

Adhering to the previous assumptions about the taxes of online orders in or outside a state with a warehouse, we could determine which states were more likely to benefit from moving a warehouse. We decided to focus on the states that had L values greater than 0.18, because there is a visible drop between the L values of North Carolina and Michigan (Fig. 8). These nine states were Michigan, New Jersey, Ohio, Pennsylvania, New York, Illinois, Florida, Texas, and California.

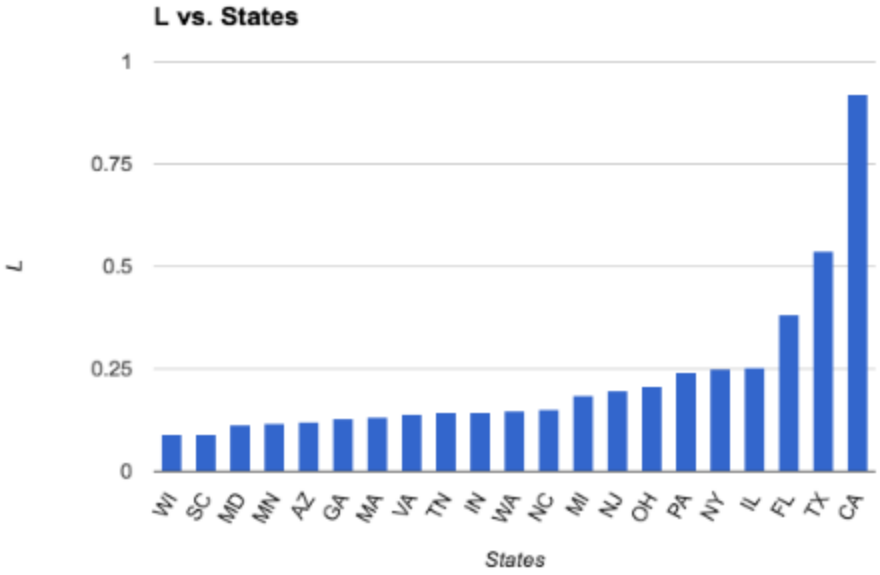

Figure 8: Graphical representation of the change in L values and states.

While rearranging the warehouses based on these new guidelines for sales tax, we did not add any extra warehouses. The cost of building another warehouse would be currently indeterminable since the amount of time needed to pay off the cost of an additional warehouse is determined by a variety of unknown factors, such as net profit of the company over time.

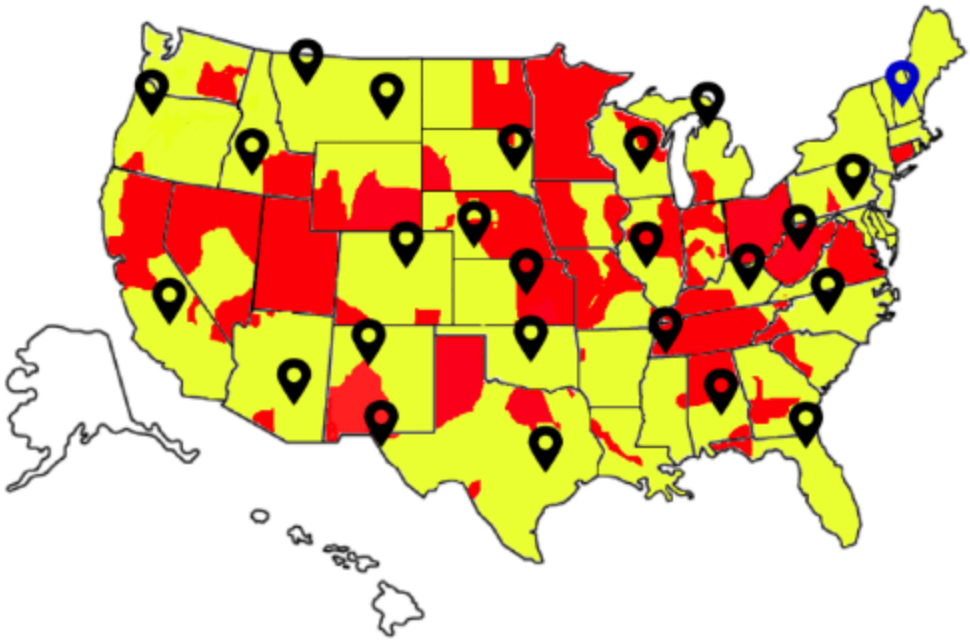

Figure 9: Map of warehouse locations altered by taxes.

The only changes in Figure 9 are the movement of the Des Moines, Iowa warehouse to Davenport, Iowa and the Hillsboro, Illinois warehouse to Evansville, Indiana. This allows us to avoid placing a warehouse in Illinois, which has the fourth highest L value, and therefore those in Illinois would now be more inclined to purchase items. Those in Indiana, however, would be less inclined to purchase items. Ultimately, Indiana's population is nearly half of Illinois so the population of Illinois has a larger impact on the L values of our model. We also move a warehouse in Memphis, Tennessee to Marion, Arkansas. Our decision to place these new warehouses were not based on our final method, as these new warehouses were usually on borders of states in order to reach surrounding states. Our method was to try ZIP codes that were both located in the center of the area we needed to cover and ones that were close to highways.

Although some states had high L values, we could not move their warehouses because those states made up the borders of the U.S. Since no other states could cover the area of the state that made up that border, we could not place a warehouse in another state because we would miss that area of the state. This statement applies to states like California, Texas, and Florida, which make up considerable portions of the national border.

Apparel Tax Liability

With the addition of apparel to the company, the overall tax liability per state changes. Now, we factor in the apparel tax into our state tax metric L, to create a modified metric L_a:

$$L_a = \frac{p_{state}}{p_{total}}(nr_a + (1 - n)r) \quad (\textit{Eq. 2: Modified tax metric per state})$$

where p_{state} is the population of the state, p_{total} is the total population of the country, n is the percent of purchases that are apparel, r is the tax rate, and r_a is the tax rate for clothing (which is either r or zero). In some states, there is no tax on clothing and shoes, which will change the tax liability of the consumers in a certain state.

An n-value of 0.3 was first used to compute L_a for each state, which resulted in some shifts in the L-rankings of the states. States with high L values that had limited apparel tax, like New York and Pennsylvania, had lower L_a values and fell in the rankings. These states became better candidates than other high L states.

Our previous n of 0.3 was an estimation by our team, but we realized that we could improve the n-value based on sales data of outdoor gear stores. Using the online shop of Great Outdoor Provision Company, we estimated the amount of apparel that was sold when compared to the total number of items sold, which was around 52.8% [6]. Although the amount of apparel sold by

different stores are varied, we tested our metric L_a with an n of .528 to see the results with a real-life n value.

Sensitivity

Our apparel tax liability model did not change from the model that just considered state sales tax liability, despite over a 20% increase in apparel out of the total number of items. While some states only moved two or three places upwards towards higher L_a values (see appendix 2), Pennsylvania and New York were the only states that significantly jumped up by seven places. We did not place any warehouses in New York and the warehouse in Pennsylvania proved to be good decision as a decrease in L_a gave those in Pennsylvania a tax reduction. This proves that our model is robust and is not heavily influenced by the varying state apparel taxes.

Strengths

- The most important facet of our model is that it allows for one-day delivery to the entire contiguous United States, while only utilizing 26 warehouses. While it is difficult to determine if this is the absolute minimum number of warehouses needed to service the entire United States, this is fewer warehouses than our first two attempts at a model required.
- Our use of a 20% range around the center of the mean ZIP code for each state attempted to mitigate this by allowing for a larger range of ZIP codes to be tested. By testing multiple ZIP codes within the range, we were able to explore more possibilities which helped our model take more possibilities into account and choose the most optimum location.
- Our model accounts for regions that are mostly uninhabited. We identified three such regions in the United States: one in Idaho, one in Maine and one in Nevada. These three regions are all isolated from the rest of the country, by forests, lack of roads and mountains respectively, and require a warehouse to be placed inside of the area to ensure one-day shipping. By recognizing that these areas typically have few to no people living in them, we are able to remove up to three warehouses which is a large improvement upon our initial plan.
- The number of warehouses our model uses is similar to the number of hubs used by Fedex, a major postal service. It is likely that the number of hubs used by the UPS is similar, indicating that our warehouses are likely near hubs, increasing the speed with which the packages can be delivered.
- The tax liability metric L is simple and easy to modify. As exemplified by the added variable of the apparel tax, the metric L could be easily modified for other variables. The metric can also be used for different areas. If the company only wanted to expand in the

East Coast, the same metric could be used by finding the total population of the eastern coast of the US.

Weaknesses

- Our final method used Utah as a starting point. The east half of the country has a higher number of roads and highways than the west half, which means that shipping locations can cover more area than in the east half. Because of this, Utah was chosen as it is the approximate center of the west half of the country. A different starting state or different states along the way could be chosen to test if it is possible to have fewer warehouses using a different starting state.
- Our model assumes that ZIP codes are always assigned east to west. This is not always the case, which means that our model excludes some ZIP codes that could have been more centered than the ZIP codes chosen for our final model. Similarly, the fact that states are rarely symmetrical means that the average ZIP codes didn't always cover the most area.
- Our model takes into account the state's fraction of the total population and the state's tax rate equally, which may not be true of how tax liability is actually calculated as companies may choose to weigh one factor more than the other.

Conclusion and Final Recommendation

We determined the minimum number and optimal placement of warehouses depending on their ZIP codes and their proximity to major roadways. The minimum number of warehouses added that would allow the 48 states to receive one-day shipping is 25. The optimal placement of warehouses can be seen in Figure 5, which only takes into account the inhabited area of the nation and leaves out the three uninhabitable parts of Idaho, Nebraska, and Maine that would require a warehouse to be placed within these uninhabited parts in order to cover it.

We found that the placement of these warehouses will affect the customer's tax liability, which we examined using a metric L that relates a state's population in comparison with the total population of the US and the tax rate of that state. By calculating L for every state, we could find which states would have the largest tax impact on its consumers. We modified L to take into account the apparel tax, and rearranged the location of the warehouses to decrease the total tax liability of the consumers. We moved the warehouse in Illinois to Indiana, the warehouse in Des Moines, to Davenport, IA, and the warehouse in Tennessee to Arkansas.

Ultimately, this plan will have to be physically implemented. Because the company currently only has one warehouse, located in New Hampshire, it is impractical to assume 25 new warehouses can be built all at once. In reality, the warehouses will have to be built one at a time.

While the new warehouses could be built east to west, gradually expanding our customer base, there is a better way.

We propose that the company build new warehouses in the following four cities: Denver, Colorado; Marion, Arkansas; Fresno, California; and Janesville, Wisconsin. Building these four bases ensures that a majority of the United States will have access to two-day shipping from the company. This will allow for a massively increased income for the company, which will allow for faster expansion as they continue to build the 21 remaining warehouses to ensure one-day shipping nationwide.

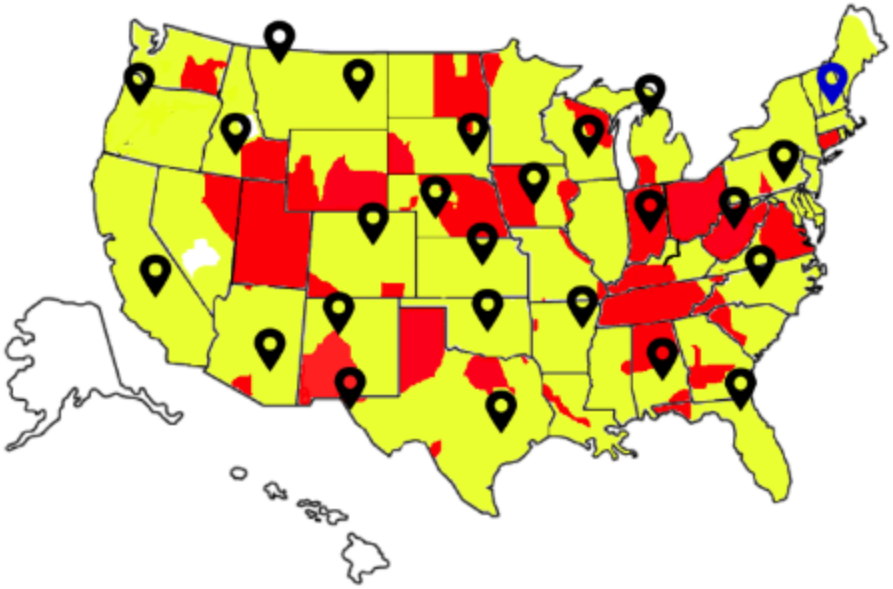

Figure 10: Final recommendation for warehouse locations.

State	Zip Code	City	State	ZIP Code	City
Florida	32068	Jacksonville	Pennsylvania	18062	Macungie
Colorado	80204	Denver	West Virginia	26104	Parkersburg
New Mexico	87107	Albuquerque	California	93722	Fresno
Montana	59832	Drummond	North Carolina	27518	Cary
Montana	59330	Glendive	Oklahoma	73119	Oklahoma City
Texas	77002	Houston	Idaho	83301	Twin Falls
Texas	79901	El Paso	Arizona	85004	Phoenix

Kentucky	40503	Lexington	Oregon	97214	Portland
Iowa	52801	Davenport	Indiana	47713	Evansville
South Dakota	57105	Sioux Falls	Alabama	36117	Montgomery
Arkansas	72364	Marion	Kansas	66606	Topeka
New Hampshire	03301	Concord	Nebraska	69101	North Platte
Michigan	49781	Saint Ignace	Wisconsin	53548	Janesville

Table 3: A list all the ZIP codes and the corresponding cities that allow for one day shipping to the entire contiguous US using our final method.

Future Work

While our work produced one method for building new warehouses across the 48 states of the contiguous United States, there are other methods that will also accomplish this. To be sure our model uses the fewest possible warehouses - or to find the method that does - our process will need to be repeated. By changing certain factors - starting on the East coast, or using South Carolina instead of North Carolina, for example - we can observe different possible outcomes to determine if this method truly is optimal. While it is hard to be certain that there are no solutions more optimal than this one, this testing would help to investigate any solutions that may be more optimal.

Using the above method, one unique case that should be investigated is building a layout around states with low tax rates. In our analysis of the problem, we altered our model to avoid states with high tax rates. However, there is only so much that can be done with this method as the arrangement is mostly set in stone. One possible way to arrive at an arrangement with lower overall tax rates is to ensure that all states with low tax rates are chosen by starting with them. This arrangement could then be compared to our final method to check 1) which is more optimal and 2) which has lower overall tax rates.

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Appendix

1) *Table 4. Tax rates, populations, and L values for each of the 48 states*

States	Tax Rate	Population	*L*	L(a) 30%	L(a) 52.8%
Delaware	0	945,934	0	0	0
Montana	0	1,032,949	0	0	0
New Hampshire	0	1,330,608	0	0	0
Oregon	0	4,028,977	0	0	0
Vermont	6	626,042	0.0118	0.0083	0.0056
Wyoming	4	586,107	0.0074	0.0074	0.0074
South Dakota	4	858,469	0.0108	0.0108	0.0108
Rhode Island	7	1,056,298	0.0232	0.0162	0.0110
North Dakota	5	756,927	0.0119	0.0119	0.0119
Maine	5.5	1,329,328	0.0229	0.0229	0.0229
Idaho	6	1,654,930	0.0312	0.0312	0.0312
Nebraska	5.5	1,896,190	0.0327	0.0327	0.0327
New Mexico	5.13	2,085,109	0.0336	0.0336	0.0336
West Virginia	6	1,844,128	0.0347	0.0347	0.0347
Colorado	2.9	5,456,574	0.0497	0.0497	0.0497
Oklahoma	4.5	3,911,338	0.0552	0.0552	0.0552
Utah	5.95	2,995,919	0.0560	0.0560	0.0560
Minnesota	6.88	5,489,594	0.1186	0.0830	0.0560
Louisiana	4	4,670,724	0.0586	0.0586	0.0586
Iowa	6	3,123,899	0.0588	0.0588	0.0588
Kansas	6.5	2,911,641	0.0594	0.0594	0.0594
Arkansas	6.5	2,978,204	0.0608	0.0608	0.0608
Alabama	4	4,858,979	0.0610	0.0610	0.0610
Nevada	6.85	2,890,845	0.0622	0.0622	0.0622
Massachusetts	6.25	6,794,422	0.1333	0.0933	0.0629

Mississippi	7	2,992,333	0.0657	0.0657	0.0657
Connecticut	6.35	3,590,886	0.0716	0.0716	0.0716
Missouri	4.23	6,083,672	0.0808	0.0808	0.0808
Kentucky	6	4,425,092	0.0833	0.0833	0.0833
Wisconsin	5	5,771,337	0.0906	0.0906	0.0906
South Carolina	6	4,896,146	0.0922	0.0922	0.0922
Maryland	6	6,006,401	0.1131	0.1131	0.1131
Pennsylvania	6	12,802,503	0.2411	0.1688	0.1138
New York	4	19,795,791	0.2486	0.1740	0.1173
Arizona	5.6	6,828,065	0.1200	0.1200	0.1200
Georgia	4	10,214,860	0.1283	0.1283	0.1283
Virginia	5.3	8,382,993	0.1395	0.1395	0.1395
Tennessee	7	6,600,299	0.1450	0.1450	0.1450
Indiana	7	6,619,680	0.1455	0.1455	0.1455
Washington	6.5	7,170,351	0.1463	0.1463	0.1463
North Carolina	4.75	10,042,802	0.1497	0.1497	0.1497
Michigan	6	9,922,576	0.1869	0.1869	0.1869
New Jersey	7	8,958,013	0.1968	0.1968	0.1968
Ohio	5.75	11,613,423	0.2096	0.2096	0.2096
Illinois	6.25	12,859,995	0.2523	0.2523	0.2523
Florida	6	20,271,272	0.3818	0.3818	0.3818
Texas	6.25	27,469,114	0.5389	0.5389	0.5389
California	7.5	39,144,818	0.9216	0.9216	0.9216
Total		318,576,557			

2) *Table 5. Rankings of the 48 states based on different L values (most tax liable to least)*

Ranking (L)	Ranking (30%)	Ranking (52.8%)
California	California	California
Texas	Texas	Texas
Florida	Florida	Florida
Illinois	Illinois	Illinois
New York	Ohio	Ohio
Pennsylvania	New Jersey	New Jersey
Ohio	Michigan	Michigan
New Jersey	New York	North Carolina
Michigan	Pennsylvania	Washington
North Carolina	North Carolina	Indiana
Washington	Washington	Tennessee
Indiana	Indiana	Virginia
Tennessee	Tennessee	Georgia
Virginia	Virginia	Arizona
Massachusetts	Georgia	New York
Georgia	Arizona	Pennsylvania
Arizona	Maryland	Maryland
Minnesota	Massachusetts	South Carolina
Maryland	South Carolina	Wisconsin
South Carolina	Wisconsin	Kentucky
Wisconsin	Kentucky	Missouri
Kentucky	Minnesota	Connecticut
Missouri	Missouri	Mississippi
Connecticut	Connecticut	Massachusetts

Mississippi	Mississippi	Nevada
Nevada	Nevada	Alabama
Alabama	Alabama	Arkansas
Arkansas	Arkansas	Kansas
Kansas	Kansas	Iowa
Iowa	Iowa	Louisiana
Louisiana	Louisiana	Minnesota
Utah	Utah	Utah
Oklahoma	Oklahoma	Oklahoma
Colorado	Colorado	Colorado
West Virginia	West Virginia	West Virginia
New Mexico	New Mexico	New Mexico
Nebraska	Nebraska	Nebraska
Idaho	Idaho	Idaho
Rhode Island	Maine	Maine
Maine	Rhode Island	North Dakota
North Dakota	North Dakota	Rhode Island
Vermont	South Dakota	South Dakota
South Dakota	Vermont	Wyoming
Wyoming	Wyoming	Vermont
Delaware	Delaware	Delaware
Montana	Montana	Montana
New Hampshire	New Hampshire	New Hampshire
Oregon	Oregon	Oregon