For office use only
T1 ________________
T2 ________________
T3 ________________
T4 ________________

For office use only
F1 ________________
F2 ________________
F3 ________________
F4 ________________

2016

19th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

(Please make this the first page of your electronic Solution Paper.)

Team Control Number: 6475

Problem Chosen: B

Please paste or type a summary of your results on this page. Please remember not to include the
name of your school, advisor, or team members on this page.

Abstract

Have you ever tried waiting forever for a UPS parcel to arrive? As the number of online stores rocketed around the world, it is getting increasingly crucial to smooth out online-offline connection. In our paper, we construct a mathematical model to determine the optimal numbers and locations of warehouses established to distribute merchandise purchased online by customers from 48 continental states in the USA.

In part 1, we assume that some particular trivial areas in some states are ignored. Then, we simplify the problem by constructing a 27-by-27 0-1 matrix. We fill the matrix with 1 or 0 according to whether the two concerned states can be mutually reached in a one-day ground shipping. On top of that, we apply Greedy Algorithm and computer analytic technique to give the optimal solution.

In part 2 and 3, we build the Vector Delivery Model. In the model, we use dots to demonstrate specific warehouse locations, and vectors to connect every warehouse with states within reach by the warehouse by one-day ground shipping. In conjunction with graph theory, we apply 0-1 programming to solve for the optimal solutions. Some adjustment by hand produces the optimal answer.

After building our model, we come up with specific answers to each question.

Key Words: one-day ground shipping, 0-1 matrix, 0-1 Programming, Greedy Algorithm, Graph Theory, optimal solution, Vector Delivery Model

Dear sir: 2016/11/13

In reference to your advertisement for a skilled sales manager, I believe that I am suitable and qualified for this post. I am writing to show my keen interest to serve in your company, and present myself as having the traits you require of a responsible, talented, and conscientious worker.

As an answer to your proposed problem in the advertisement, I would like to present my recommendation of warehouse locations to maximize benefit for your distribution company. Hopefully it will turn out to be yet another cornerstone for your company's prosperous future.

Please let me restate the problem. Your company is now pondering on expanding business to the entire continental United States, and is currently building a desirable blueprint for warehouse establishment across the continent. Your requirement is that the plan should minimize the number of warehouses and reduce sales tax for various sorts of commodities. The following is my suggestion and why I propose my advice.

According to the information you provide, we start off to consider how to exclusively minimize the amount of warehouse establishment. The final results our team is able to produce is that the warehouse location which comprises minimum warehouse quantity is (**KY, MI, MN, MO, NE, NY, NC, PA, WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL),** where the state that precedes "(2)" should possess 2 warehouses to cover the entire region of this state. In total, we determine 26 warehouse locations out of 48 states of the United States. Our team applies various mathematic approaches to address the issue. Through the agency of computer programming, which we keep on modifying until it outputs the most desirable answer, we are able to determine the ultimate and optimal locations of warehouses. In order that you will have other plans as alternatives, we will also render other two of our plans in this letter. They are (**NE, KS, MN, IL, OH, KY, NY, PA, NC, WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL),** and **(IA, WI, NC, MO, MI, NY, KY, MN, PA ,WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL).** The advantage of this plan is that it creates a smooth online-offline connection, and can deliver goods to any given point in the continental United States within one day. That will greatly satisfy your clients and hike people's interests in your service. In addition, this plan will reduce the cost of your company because it can greatly reduce the cost of establishing warehouses, for it minimizes the total number of them.

We go ahead to further our discussion and consider when sales tax is added. Theoretically, the solution we produce should also ensure minimum amount of warehouses. Our plan is (**NE, MN, MO, MI, KY, WV, SC, DE, VT**, **WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL)** for when every state requires a proportion of sales as tax, and (**MO, NE, MN, MI, KY, SC, NY, PA, WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL)** for when clothes tax is considered exclusively. We surprisingly found that the number of warehouses do not increase significantly. To come up with this solution, we consider various variables such as digital shoppers in each state, to found our plan on a more realistic basis. We apply computer programming to respectively solve for two plans. In order to establish the answer as the most optimal, we are mindful of making some adjustment by hand. This plan, on top of the advantage mentioned above, will augment demand of your service. Since customers are appropriately taxed, they are more likely to take your company's delivery as the optimal choice. This preference will significantly expand your company's influence in the United States and even the world, and earn your company renown as being extremely competitive.

I would be grateful if you arrange for an interview with me. I have long appreciated your company's ambition, and hopefully I can become a part of the challengers myself. Thanks for taking time for reading this letter. I most sincerely wish my plan is workable for your company.

Yours sincerely

xxx

2016/11/13

TEAM #6475

OPTIMAL WAREHOUSE LOCATIONS: WHERE DO THE GOODS GO?

Contents

1.Introduction

As online stores gain its popularity across the globe, the traditional brick and mortar stores are challenged to reevaluate and reposition itself in the growing favor for its counterpart. The problem arises: how to manage the locations of physical stores (or warehouses) in order to efficiently smooth out online-to-offline connection? It will be beneficial for both customers and sellers to be sufficiently acknowledged in this aspect, for through online sales and offline delivery, sellers are able to minimize prime cost and customers will be freed from having to travel to physical stores for purchase. In this paper we present a mathematical model for determining optimal locations for warehouse settlement under various types of circumstances. The model includes a plan to minimize the number of warehouses, a plan to minimize the general tax liability for customers across the USA, and a plan to optimize clothes expenditure for each buyer. In addition, we also wrap up a letter to the CEO of our company. We believe our plans are able to bring convenience to all who are concerned.

2. Assumptions and Basic Analysis

2-1 Assumptions:

(1) *Every state's radius of UPS delivery is given by this state's capital's zip code.* Since most of a state's online purchase population gather around the city's capital, setting warehouses there is more favorable and desirable in terms of convenience of delivery to local customers and shipping to other adjacent states. In this regard, under the condition that the capital of the state can reach the entire state in one day, warehouse is taken to be in or near the capital.

(2) *When determining whether a state can be fully reached, we ignore some trivial areas that are so remote that other states cannot reach*. If the state can be covered except these special areas, we take these states as fully covered. In the US, and many parts of the world, some areas are so remote that they do not have decent delivery route or stations for goods distribution. In order to secure company's interest and procure as much benefit as possible, parcel delivery service requires that the people living in these areas travel to a place within reach by delivery to acquire their purchase. In general, we are able to point out a few of these areas:

①the North-Eastern part of Maine (the one in brown)

Fig. 1

②the Northern part of Michigan. (the one in brown)

Fig. 2

(3) *We take the population of a state to predict a state's population of digital shoppers. Furthermore, we assume the number of a state's digital shoppers in right proportion to its population.(the ratio is the proportion of number of digital shoppers in the entire US population). So for the sake of our problem, we use a state population to indicate the state's population of digital shoppers.* America is a highly developed country, and all states generally remain abreast in terms of economy. We thus deem that all states have an equal ratio of digital shoppers. Survey suggests online shoppers tend to live in households with higher-than-typical incomes. An Experian survey found that 55% of e-commerce shoppers in the U.S. live in households with incomes above $75,000(40% were in households earning $100,000 and above). The median household income in the U.S. is around $50,000, according to the Census. [1] So we use the general ratio of people whose income are higher than average in the US, to demonstrate the same ratio in each state. After calculating according to the graph as follows, the ratio is set to be 58.5%.

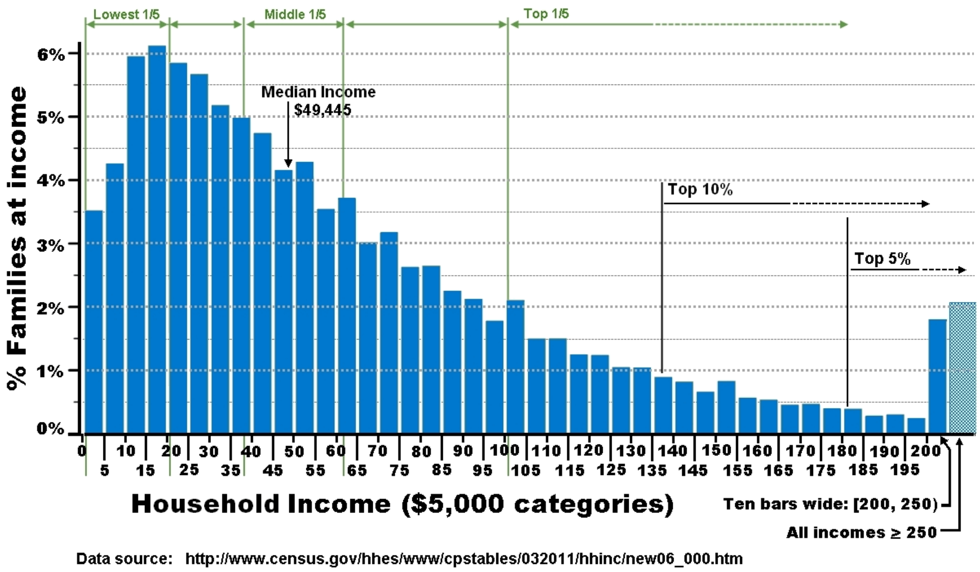

Fig. 3 Household Income distribution

2-2 Basic Analysis:

(I)A brief model overview:

We are required to come up with a plan of warehouse locations to meet the demands of the entire continental United States (comprising 48 states). Our aim is to make the plan as workable and efficient as possible. A well-built model should:

1. make sure that warehouses as a whole are able to distribute goods to everywhere in the continental USA in one day.
2. reduce the number of warehouses as many as possible
3. reduce the tax liability of customers.
4. provide a precise layout of specific warehouse locations.

(II)Facts we should know:

Fact 1. Many western states in America possess large territory, such as the state of California, one warehouse may not be sufficient to cover the entire state and meet up its demand. We will allocate two warehouses in one state in this case. Also, because of the uniqueness of geographic locations, many states in western and central United States are only able to supply itself in one-day ground shipping. For the sake of our paper, we name these states as ***Independent States***. Since ***Independent States*** are totally independent in terms of setting warehouses, we will ignore these states in the following discussion.

Also, we find that some states possess the following nature: (1) they are mutually reachable in one-day ground shipping. (2) They cannot reach other states except each other and themselves. Then we name these states as ***Bi-dependent States.*** In this case, we only need to place one warehouse to meet the demands of this region.

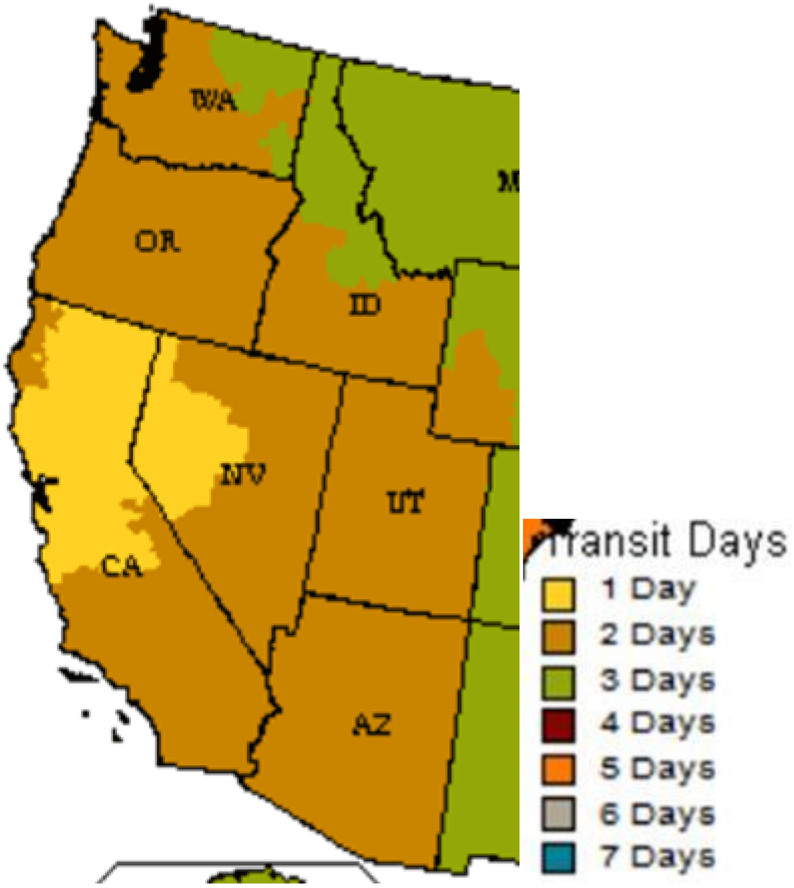

Fig.4 one-day delivery from California (two warehouses)

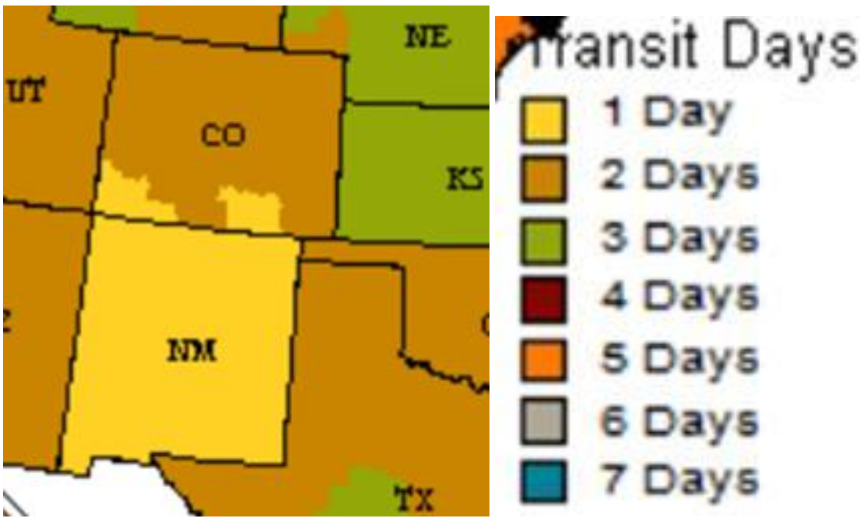

Fig.5 one-day delivery from New Mexico (Independent State)

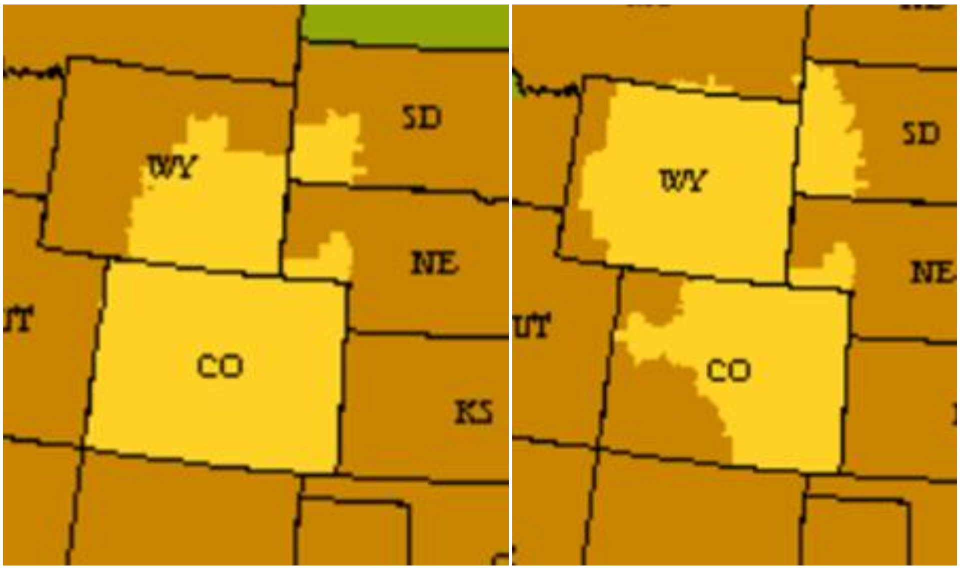

Fig.6 One day delivery from Colorado and Wyoming (Bi-dependent States)

Fact2. ***Independent States*** and ***Bi-dependent States*** are AL, AZ, AR, CA, CO, FL, GA, ID, LA, MS, MT, NV, NM, ND, OK, OR, SD, TX, UT, WA, WY. After "breaking" them from the American territory, the whole map of America will look like:

Fig.7 Leftover states when Independent States and Bi-dependent states are taken out

Fact3. Some states with warehouses can only reach a limited region of another state. To address the problem, in our plan in Part I, we will highlight this relationship with a red "1" if the region within reach takes up the chunk of the state's area (namely between one half and two thirds) We will carefully go through the process of selecting ultimate warehouse locations in the agency of this specific red mark.

(III) Notations and State rank:

(i) State rank:

For the sake of our problem solving, we rank the state as follows:

State Rank

NO.1 CT	NO.10 MA	NO. 19 OH
NO.2 DE	NO. 11 MI	NO. 20 PA
NO.3 IL	NO.12 MN	NO. 21 RI
NO.4 IN	NO. 13 MO	NO.22 SC
NO. 5 IA	NO.14 NE	NO.23 TN
NO.6 KS	NO. 15 NH	NO. 24 VT
NO.7 KY	NO.16 NJ	NO. 25 VA
NO.8 ME	NO. 17 NY	NO.26 WV
NO.9 MD	NO.18 NC	NO.27 WI

Fig. 8 State Rank

(ii) Notations:

n_i : tax rates for each state in the continental United States, where $i \in \{1,2,3...27\}$ and is given according to the state's state rank. For example, the tax rate for the state of Connecticut is denoted as n_2.

a_i : number of states one state can reach in a regular UPS one day ground shipping service. Note that $i \in \{1,2,3...27\}$ and is given in the same way as in n_i.

p_i : the number of households with higher-than-average income, (the predicted population of online shoppers in each state).

T_s : denotes the total tax liability of customers across the continental United States.

b_{ij} : the number of overlapping states of two states i and j.

x_i : used in Part 2 to determine whether a state has warehouse or not. If the state has warehouse(s), $x_i = 1$; If not, $x_i = 0$.

v_i : a set of states that satisfy the following nature: (1) the states within the set cannot reach states outside of the set. (2) States outside the set cannot reach the states within the set.

3. Mathematical Modeling:

3-1 Plan of optimal warehouse locations Part 1:

(I) The construction of the *0-1 Matrix* :

In this plan, we concern ourselves with the task of determining the least number of warehouses established in the United States, and their specific locations.

We define the following matrix as the *0-1 Matrix*: The matrix is filled with number 0 and 1. When the delivering state, or the state who owns a warehouse, can reach another state in one day by ground shipping, then we mark the according position in the matrix as "1", otherwise, we denote it as "0". Red "1"s are marked according to the plan proposed in **Fact3**.

After an arduous process of synthesizing data, we attain a matrix as follows: (Since matrix cannot illustrate states clearly, we turn to Excel file to achieve the same objective.) [2]

	CT	DE	IL	IN	IA	KS	KY	ME	MD	MA	MI	MN	MO	NE	NH	NJ	NY	NC	OH	PA	RI	SC	TN	VT	VA	WV	WI
CT	1	1	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	0	0	1	1	0	0	1	0	0	0
DE	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	1	0	0
IL	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
IN	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
IA	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
KS	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
KY	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0
ME	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	0
MD	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0
MA	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	0	0	0	1	0	0	1	0	0	0
MI	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
MN	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MO	0	0	1	0	1	1	0	0	0	0	0	0	1	O	0	0	0	0	0	0	0	0	0	0	0	0	0
NE	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
NH	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0	1	0	0	1	0	0	0
NJ	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0
NY	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0
NC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0
OH	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
PA	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
RI	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	0	0	0	1	0	0	1	0	0	0
SC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
TN	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
VT	1	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	0	0	0	1	0	0	1	0	0	0
VA	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	1	0
WV	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0
WI	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Fig. 9 0-1 Matrix for 27 states

(II) Application of the 0-1 Matrix to solve by computer:

Our first step in this process will be: taking all red “1”s as black “1”s, and going through a selection process by a computer as follows:

①Observe each line of the matrix, and select the state with least “1”s in it.

②Starting from this state we will have at least one state as its deliverer of goods.

③We go through each situation and if we have determined a warehouse’s location, we will delete its belonging state from the Matrix, meaning both its according row and line will be erased from the matrix.

④If the left-over matrix is empty, then we collect our results of selection by computer; If not, we will continue the program and repeat the upper three steps until it ends.

We use computer to solve for answer (code see Appendix) .

The results are, in total 27 possibilities:

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	3	6	8	10	12	14	16	19	0	0	0
2	2	3	6	8	10	12	15	17	20	0	0	0
3	2	3	5	7	9	11	14	16	19	0	0	0
4	2	4	5	7	9	12	14	17	19	21	23	0
5	2	4	5	7	9	11	14	16	19	0	0	0
6	2	4	6	7	9	12	14	16	19	0	0	0
7	2	4	6	7	8	10	13	15	18	0	0	0
8	2	4	7	8	9	11	13	15	18	0	0	0
9	3	6	8	9	10	12	15	17	20	0	0	0
10	2	4	7	9	10	11	13	15	18	0	0	0
11	2	4	7	9	11	13	17	20	0	0	0	0
12	2	4	7	9	11	12	14	16	19	0	0	0
13	2	4	7	10	12	13	14	16	19	21	23	25
14	2	4	7	9	12	14	16	18	21	0	0	0
15	2	4	7	9	11	13	15	18	0	0	0	0
16	3	6	8	10	12	14	16	20	0	0	0	0
17	2	4	7	9	11	13	15	17	20	0	0	0
18	2	4	7	9	11	13	16	18	0	0	0	0
19	2	4	6	8	10	14	17	19	0	0	0	0
20	3	6	8	10	12	15	17	20	21	0	0	0
21	2	4	7	9	11	13	15	18	21	0	0	0
22	2	4	7	9	11	13	16	18	22	0	0	0
23	2	4	7	9	11	14	16	19	21	23	0	0
24	2	4	7	9	11	13	15	18	24	0	0	0
25	3	6	8	10	12	15	17	19	21	25	0	0
26	2	4	7	9	11	14	16	19	21	26	0	0
27	2	4	7	9	11	13	16	18	21	27	0	0
28												

Fig. 10 Part 1 results

Note: numbers filled in the units are State Ranks.

Now we have gotten the results. If we put it in practice and find that the solution is workable and all continental America can be covered, then we must have gotten the ultimate answer. This is because we have considered red “1”s as black “1”s, meaning that we have used a supposedly larger area to cover the USA in order to get the solution. If we now shrink this area into its original one, and it still can achieve our aims, then the warehouse locations given is the optimal one. However, if the solution is in effect not workable, a possible ramification is that we will not be able to cover the entire United States using these warehouses. We will make amends in the following step.

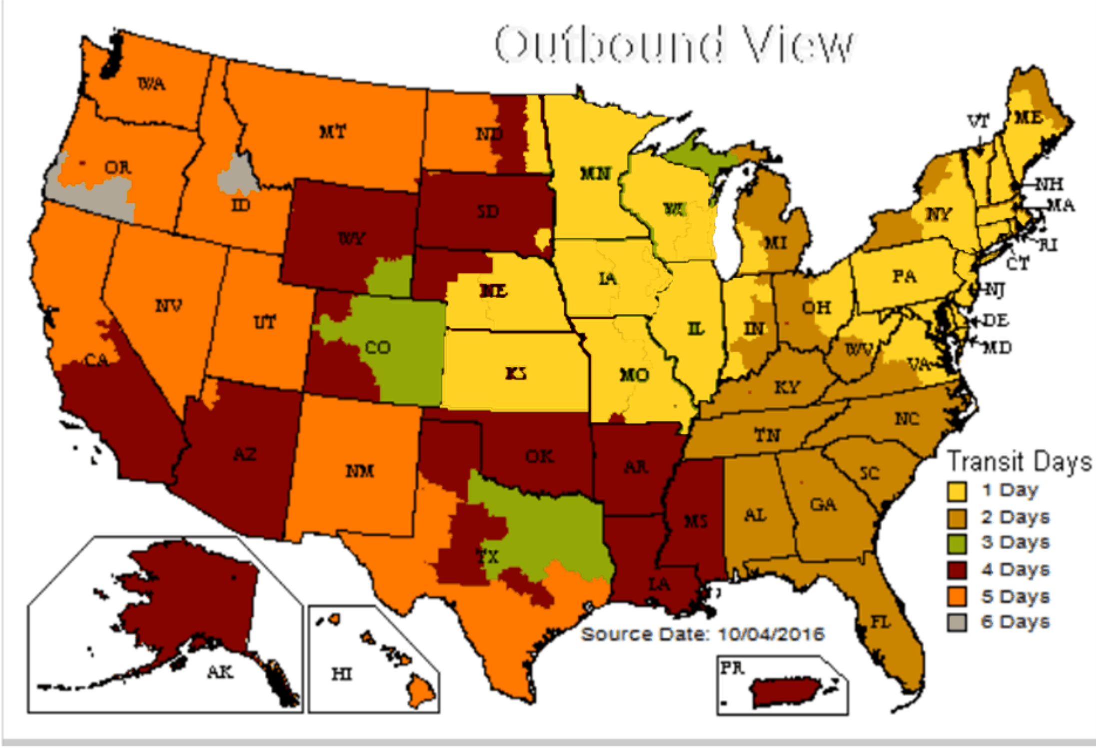

Fig.11.1 some alternatives cannot cover the entire left-over United States

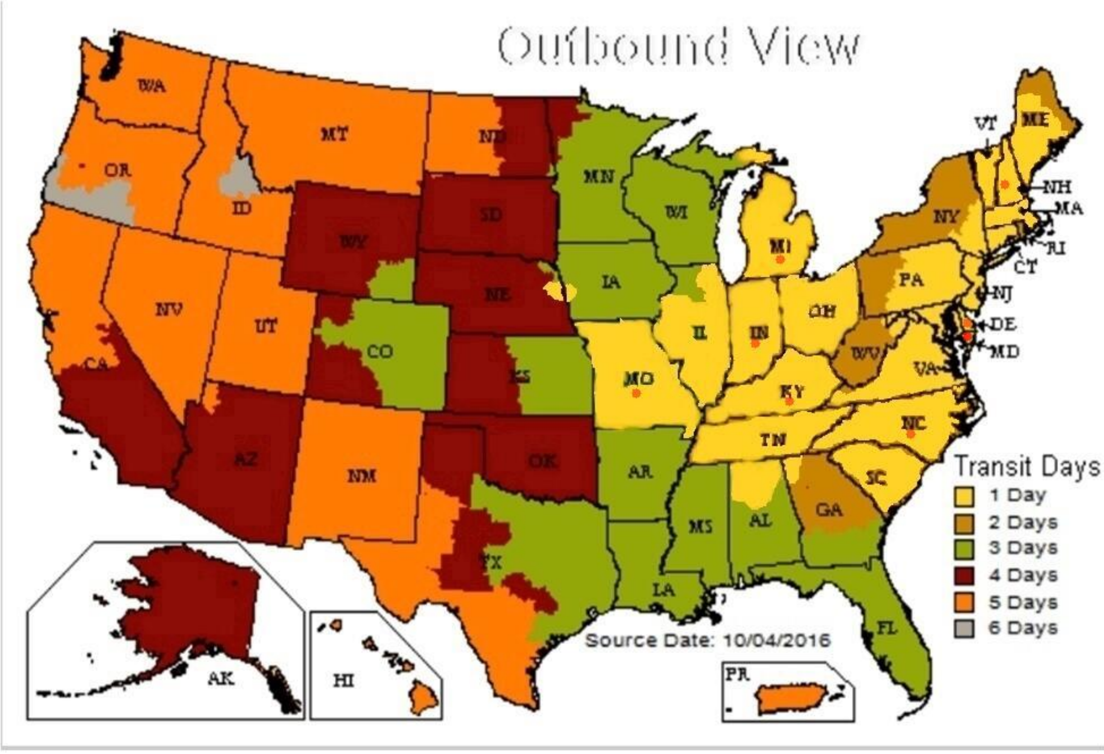

Fig. 11.2 some alternatives cannot cover the entire left-over United States

(III) Modification of method in (II):

One possible practice is that for each solution we calculate the red "1"s we have gone through in the process of solving in (II), and choosing from the set of solutions (given in (II)) the least number of warehouses with the least number of red "1"s. In general, we try to find the best possible plan where the area we use to determine the solutions is shrunk as little as possible, and therefore more likely to satisfy our demands.

Another possible way is that we put in practice every alternative given in (II), starting from the one with the least number of warehouses, and find the first set of solution that can satisfy our demands. During the process, we can adjust or add new warehouses to cover areas as large as possible.

Then the solutions are:

a. KY, MI, MN, MO, NE, NY, NC, PA

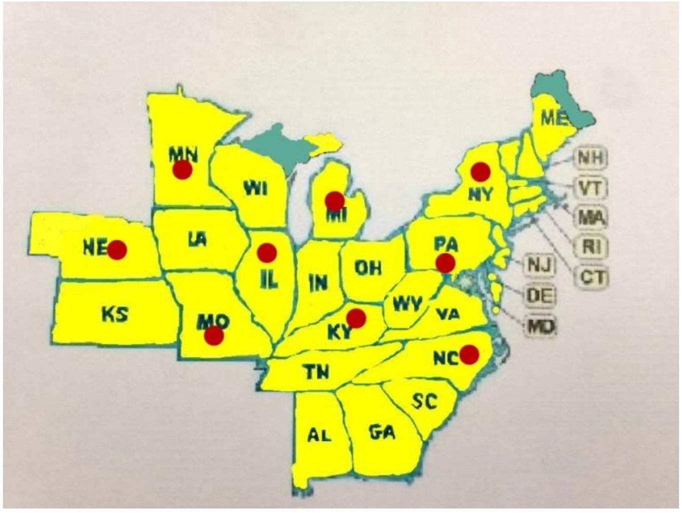

Fig. 12.1 first set of solution for Part 1

b. NE, KS, MN, IL, OH, KY, NY, PA, NC

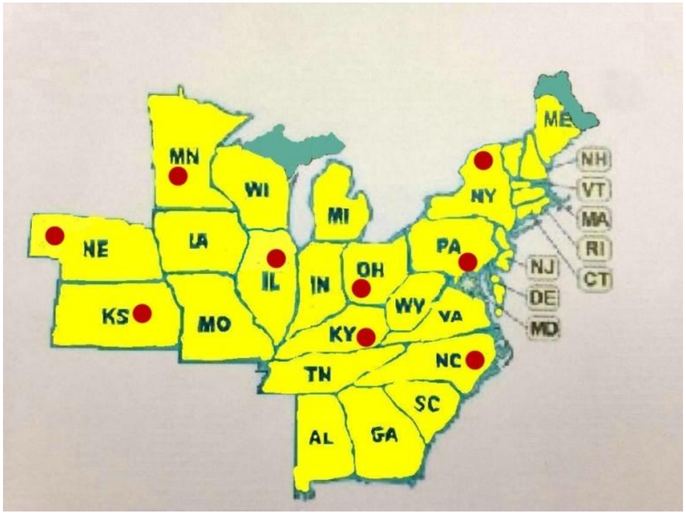

Fig. 12.2 second solution for Part 1

c. IA, WI, NC, MO, MI, NY, KY, MN, PA

In comparison, the three plans all will minimize the number of warehouses (9 in 27 states), but **c** cannot cover as many states as **a** and **b** do.

3-2 Plan of optimal warehouse locations Part 2:

(I) Impact of tax liability to solutions given in Part 1:

We calculate the tax liability of possible locations of warehouses in Part1. We find：

a. The total tax liability for the 27 states is 441.4
b. The total tax liability for the 27 states is 441.8

c. The total tax liability for the 27 states is 389.2

Though c does not seem like a desirable plan in part 1, sales tax can be greatly reduced if we opt c in part 2, which impacts our choice in Part 1.

So we will continue to discuss this problem when more variables are added.

(II) New solutions given if tax liability is considered:

Step.1 Constructing the *Vector Delivery Model* based on graph theory*:*

As provided by the problem notes, we are able to determine each state's tax rates as follows:

State Sales Tax Rates for 48 Continental US States

AL 4%		AZ 5.6%	AR (2) 6.5%
CA 7.5%	CO 2.9%	CT 6.35%	DE (1) No Sales Tax
DC 5.75%	FL 6%	GA 4%	
ID 6%	IL (2) 6.25%	IN 7%	IA 6%
KS 6.5%	KY 6%	LA 4%	ME 5.5%
MD 6%	MA(1) 6.25%	MI 6%	MN (1) 6.88%
MS(2) 7%	MO 4.23%	MT (1) No Sales Tax	NE 5.5%
NV 6.85%	NH (1) No Sales Tax	NJ (1) 7%	NM 5.13%
NY (1) 4%	NC 4.75%	ND 5%	OH 5.75%
OK 4.5%	OR (1) No Sales Tax	PA (1) 6%	
RI (1) 7%	SC 6%	SD 4%	TN (2) 7%
TX 6.25%	UT (2) 5.95%	VT (1) 6%	VA 5.3%
WA 6.5%	WV 6%	WI 5%	WY 4%

(1) None or limited tax on clothing and shoes.
(2) Some tax on groceries.

Fig. 13 Sales tax for each state

Ignoring ***Independent States,*** we think of constructing a ***Vector Delivery Model*** that satisfies:

(1) If one delivering state can reach another state, then we draw a vector with arrow pointing from the delivering state to the receiving state.

(2) If one delivering state cannot reach another state, then we do not draw the vector.

If we apply this model to the state of West Virginia, if OH, KY, VA all possess warehouses, the regional map will look like:

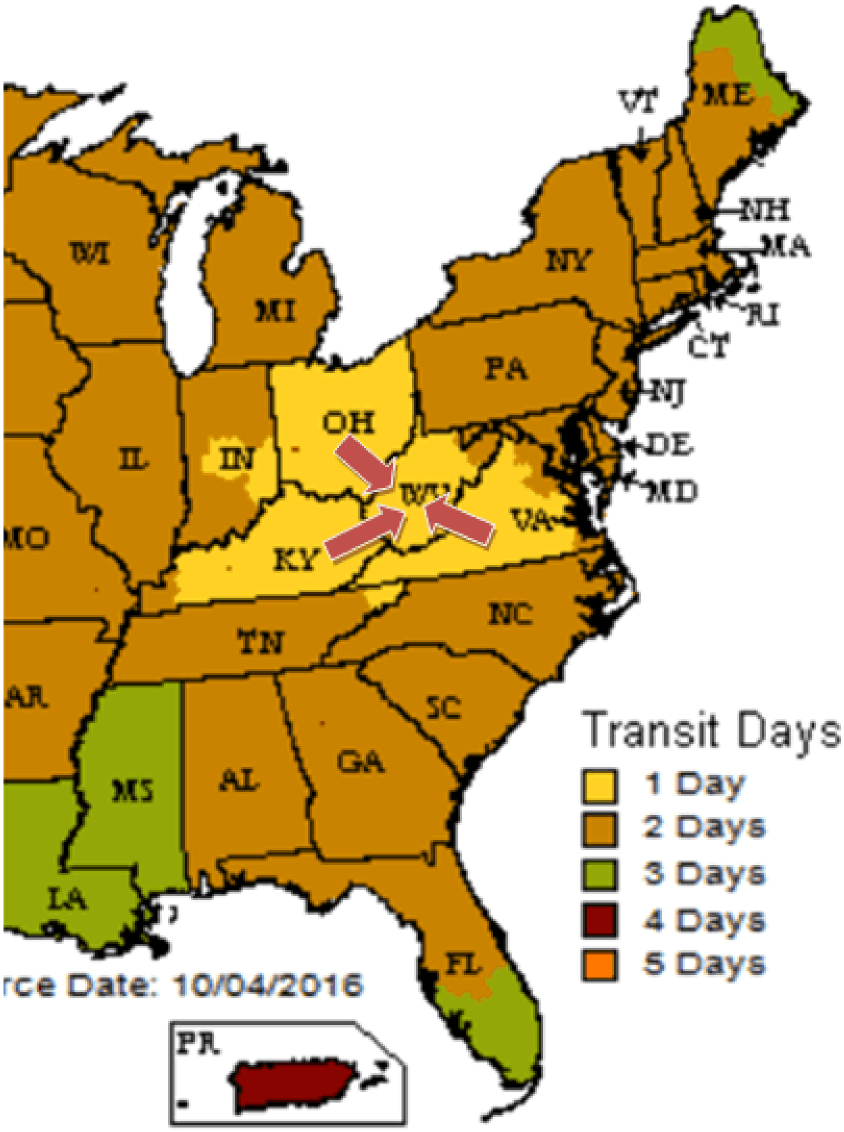

Fig.14 Vector Delivery Model of West Virginia

Step.2 Linear Programming:

Now we start to consider using n_i, a_i, p_i.

Since we know our aim is to minimize average tax liability of the American customers, we start to consider the minimum of the following expression:

$$T_s = \sum_{i=1}^{27} x_i p_i n_i$$

Now it is necessary for us to calculate a_i, p_i for each state:

	CT	DE	IL	IN	IA	KS	KY
a_i	10	6	3	4	5	3	5
	NE	NH	NJ	NY	NC	OH	PA
a_i	3	6	9	9	3	4	7
	ME	MD	MA	MI	MN	MO	
a_i	6	5	8	2	3	2	
	RI	SC	TN	VT	VA	WV	WI
a_i	6	2	2	7	5	4	2

Fig.15 a_i ***for each state***

CT	DE	IL	IN	IA	KS	KY	ME	MD	MA	MI	MN	MO	NE
359.6	92.5	1288.2	657	309	289.3	439.5	132.8	592.8	669.2	989.5	542	604.4	186.8
NH	NJ	NY	NC	OH	PA	RI	SC	TN	VT	VA	WV	WI	
132.3	889.9	1965.1	984.8	1157	1277.3	105.1	477.4	649.5	62.6	826	185.4	574.2	

Fig.16 p_i ***for each state*** [3]

	CT	DE	IL	IN	IA	KS	KY	ME	MD	MA	MI	MN	MO	NE	NH	NJ	NY	NC	OH	PA	RI	SC	TN	VT	VA	WV	WI
CT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DE	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IN	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IA	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
KS	0	0	1	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
KY	0	0	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ME	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MD	3	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MA	8	2	0	0	0	0	0	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MI	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MN	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MO	0	0	1	0	2	2	0	0	0	0	0	0	0	O	0	0	0	0	0	0	0	0	0	0	0	0	0
NE	0	0	0	0	3	2	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NH	6	1	0	0	0	0	0	6	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NJ	8	5	0	0	0	0	0	4	4	6	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0
NY	9	3	0	0	0	0	0	6	2	8	0	0	0	0	6	7	0	0	0	0	0	0	0	0	0	0	0
NC	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
OH	0	0	1	3	0	0	3	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PA	4	6	0	0	0	0	1	1	5	2	0	0	0	0	1	5	3	1	0	0	0	0	0	0	0	0	0
RI	6	1	0	0	0	0	0	6	0	6	0	0	0	0	6	4	6	0	0	1	0	0	0	0	0	0	0
SC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
TN	0	0	0	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
VT	7	1	0	0	0	0	0	6	0	7	0	0	0	0	6	5	7	0	0	1	6	0	0	0	0	0	0
VA	2	3	0	0	0	0	1	0	3	0	0	0	0	0	0	2	1	2	0	4	0	1	0	0	0	0	0
WV	0	1	0	2	0	0	3	0	1	0	1	0	0	0	0	0	0	1	2	2	0	0	1	0	2	0	0
WI	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Fig.17 $b_{i,i+j}$ ***for each state***

Now we consider the following linear programming problem:

$$\min T_s = \sum_{i=1}^{27} x_i n_i p_i$$

$$s.t \qquad \sum_{i=1}^{27} a_i x_i \geq 27\,; \qquad ①$$

$$\sum_{i=1}^{27} a_i x_i - \sum_{i,j\in\{1,2..27\},i+j\in[1,27]} b_{i,i+j} x_i x_{i+j} \leq 27 \qquad ②$$

$$x_i = 0,1$$

We now explain why ① and ② are true and strong enough.

For ①, we consider the set of states v_i, its definition given in Notations.

We know that if there exists such set, then the warehouse number in this region should also satisfy something in the form of $\sum_{i=1}^{k} a_i x_i \geq k$, and will add another restraint to our linear programming.

We start from the state Nebraska, and conduct the following selection process:

①find the states-within-reach by one-day ground shipping from the chosen state, and add to the set.

②find the states-within-reach by one-day ground shipping from each of the newly added states (repeat step ①)

In the case of this specific problem, the set expand as shown in the following:

(NE), (NE, KS, IA), (NE, KS, IA,MO,MN), …

Then we find that all 27 states are included in the same set, meaning that our original qualification is strong enough.

For ②, we consider the number of states in the overlapping area of two groups of states reachable by two states $i, i+j$ respectively, and we agree that this number is $b_{i,i+j}$.

If two states both own warehouses, when we calculate the number of states the two states cover, we have to subtract the according $b_{i,i+j}$ from the sum of a_i and a_{i+j}. However, this will give us a smaller number than 27, for if there exist three states overlapping, we will subtract more than is necessary.

Therefore, to determine whether both states own warehouses, we calculate $x_i x_{i+j}$. If the expression equals 1, both states possess warehouses; if the expression equals 0, but all two states own warehouse(s).

The program code for solving the problem is provided in the Appendix [4-6]:

The output results are:

a. IL, IA, KS, MA, NC, OH, SC, TN

b. IL, IN, MA, NC, OH, PA, RI, SC, WV

c. IL, KS, MA, MV, NC, OH, RI, SC, TN, WV

d. ME, MA, MI, NH, NC, WI, OH, PA, SC, TN, VA

But since we only have considered regional optimal choices, we discover, by observing from the four sets of results, DE, VT are the most optimal choices, and KY plus MI is better that OH plus TN. We are able to finally come up with a relatively optimal choice:

NE, MN, MO, MI, KY, WV, SC, DE, VT, where the sum of tax for the 27 states is 202.4. As shown in the figure bellow, this plan also minimizes the amount of warehouses established. In general, we have achieved both aims by this plan.

Fig.18 Part 2 solution on the map

3-3 Plan of optimal warehouse locations Part 3:

(I)Impact if clothes tax is considered in Part 1 and 2：

CT	DE	IL	IN	IA	KS	KY
6.35%	0	6.25%	7%	6%	6.50%	6%
NE	NH	NJ	NY	NC	OH	PA
5.50%	0	0	0	4.75%	5.75%	0
ME	MD	MA	MI	MN	MO	
5.50%	6%	0	6%	0	4.23%	
RI	SC	TN	VT	VA	WV	WI
0	6%	7%	0	5.30%	6%	5%

Fig.19 n_i ***for each state when considering clothes tax***

Part 1:

a: The total tax levied in the 27 states is: 168.3

b: The total tax levied in the 27 states is:249.2

c: The total tax levied in the 27 states is: 205.3

Part 2:

The total number of tax levied in the 27 states is: 161.3

(II)New arrangement of warehouse locations:

We come up with a new form of n_i. As shown in Fig. 9.

Inputting this new set of n_i, and we go through the same process presented in Part 2.

The according code is provided in the Appendix.

We get a satisfactory result: **MO, NE, MN, MI, KY, SC, NY, PA**, where tax liability is determined to be 150. As shown in the figure bellow, this plan comprises only eight warehouses. However, it is observable that gaps still exist in several notable places.

Fig. 20 Part 3 solution on the map

4. Results:

4-1 Plan 1:

There are three possible plans, we rank them from one to three by the criteria of how well they can cover the US territory.

1. **KY, MI, MN, MO, NE, NY, NC, PA, WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL**

2. **NE, KS, MN, IL, OH, KY, NY, PA, NC, WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL**

3. **IA, WI, NC, MO, MI, NY, KY, MN, PA ,WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL**

4-2 Plan 2:

There is one optimal plan:
NE, MN, MO, MI, KY, WV, SC, DE, VT, **WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL**

4-3 Plan 3:

There exists an optimal plan:
MO, NE, MN, MI, KY, SC, NY, PA, WA, OR, CA(2), UT, AZ, MT, NM, WY, ND, SD, OK, TX(2), AR, LA, GA, FL

5. Discussion: Strengths and Weaknesses:

5-1 Strengths:

(1) We apply the methodology of **0-1 programming** in many aspects of our model. We construct an associated 0-1 matrix to demonstrate whether any given two states are related in a regular UPS service. We also utilize 0-1 planning to solve our problems in Part 2 and 3. This method of 0-1 planning can be effectively used to make connections between graph theory problems and linear (or nonlinear) planning problem.. The model is appropriate and highly applicable because 0-1 simplifies the graph as a whole. In addition, the simplified version of our problem is amenable to computer programming. The help of computers have rendered our answers accurate and precise. [7]

(2)**Greedy Algorithm** is used in our solving in Part 1. We opt states most likely to be the final solution, and calculate whether that is real. With *0-1 Matrix*, we are able to come up with a program that eventually leads to the optimal solution set.

(3) We apply **Graph theory** to simplify the map as a whole. Our *Vector Delivery Model* has so greatly simplified the problem that our subsequent methods become easy to be programmed by computers.

(4) Our approach in Part 2 and 3 can be widely used to address problems of the same kind. For instance, setting warehouses around the world is also amenable to this approach. As long as we have data in hand, with the help of computer programming, we are able to eventually come up with an optimal solution.

5-2 Weaknesses:

(1) Our approach in Part 1 is applicable when addressing the entire United States. However, it is no longer the optimal method when addressing larger and more complex areas such as Asia, or the entire world. This is because, for the US, we have used computers to simulate every possible outcome. In the case of the world, we are challenged to simulate more than 10^{13} possibilities. Unless our approach is modified, this mind-boggling number will not be addressed easily.

(2) In our model, we apply the population of each state as a key index to predict the population of digital shoppers in each state. However, this is not always true. It is better if we can take into account different types of variables, such as average GDP in each state, and number of higher-than-typical households.

6. Reference

[1] originated from http://www.census.gov/retail/mrts/www/data/pdf/ec_current.pdf

[2] Lu Jianl, Yang Mingbo (2005). The Realizing of Matrix in Graph Theory， Mathematics in Practice and Theory, Vol 35, 453007.

[3] originated from http://wapwenku.baidu.com/view/d7a1ed1902d276a201292e2f.html?ssid=0&from=1012852q&uid=0&pu=usm@1,sz@1320_1004,ta@iphone_2_4.4_11_8.2&bd_page_type=1&baiduid=F6D6F83142F145AF29C756D9FBAFDCC7&tj=wenkuala_3_0_10_l1#1

[4] originated from http://wenku.baidu.com/link?url=cX2RCysUufKTEvY6V02gNax35504_Cyqz5eFqPfo67ryrFToSaxKhaj22au5NjECW2Kx-n2uxG35SYWWl5OsiS4R8w8pHGKcanAA62zWJoq

[5] originated from http://blog.sina.com.cn/s/blog_6622f5c30100l97z.html

[6] originated from http://blog.chinaunix.net/uid-8867796-id-358759.html

[7] Chen Huayou, Zhou Ligang, Liu Jingpei, (2014). 《Mathematics Modelling》. Science Press (Chinese Science Publishing & Media Lsd.)

7. Appendix

7-1 Part 1

```
% Num_matrix=zeros(27,1);
% for i=1:27
%   Num_matrix(i,1)=length(find(matrix0(i,:)~=0))
%   min_matrix= find(Num_matrix==min(Num_matrix))
%   for j=1:length(min_matrix)
%       Num_fugai(j,:)=find(matrix0(min_matrix(j,1),:)~=0)
%       for k=1:length(Num_fugai)
% end



%        for j=1:27
%            if
length(find(matrix1(j,:)~=0))~=0|length(find(matrix1(:,j)~=0))~=0
%                route(i,j)=1;
%                l(i,1)=l(i,1);
%                break;
%            end
%        end

%%%%%%%%%%Question1
matrix0=xlsread('Book2');
k=zeros(27,1);
route=zeros(27,27);
for i=1:27
l=0;
    while length(matrix0)~=0
        Num_l1=find(matrix0(1,:)~=0);
        matrix0(Num_l1(:),:)=[];
        matrix0(:,Num_l1(:))=[];
        route(i,i)=1;
      %  k(i,1)=k(i,1)+1;
        j=1;
        while j<=(length(Num_l1)+1)
            if j==length(Num_l1)+1
                l=l+j;
                route(i,l)=1;
```

```
            else
                if Num_ll(j)~=j
                    l=l+j;
                    route(i,l)=1;
                    break;
                end
            end
            j=j+1;
        end
    end
    matrix0=xlsread('Book2');
    if i~=27
        matrix0([1,i+1],:)=matrix0([i+1,1],:);
        matrix0(:,[1,i+1])=matrix0(:,[i+1,1]);
    end
end
route1=zeros(27,12);
for i=1:27
     route1(i,1:length(find(route(i,:)~=0)))=find(route(i,:)~=0);
end
```

7-2 Part2

```
%
f=[0.635,0,0.1875,0.28,0.18,0.26,0.30,0.385,0.36,0.50,0.12,0.1376,0.1
692,0.165,0,0.49,0.20,0.1425,0.2875,0.24,0.56,0.12,0.14,0.48,0.318,0.
24,0.10];
% A=[10,7,3,4,3,4,5,7,6,8,2,2,4,3,7,7,5,3,5,4,8,2,2,8,6,4,2];
% A=-A;
% b=-27;
%  [x,fv,ex]=bintprog(f,Ai,b,[],[])



%%%%%Question 2
Pop=xlsread('population.xlsx');
Ni=xlsread('ni.xlsx');
Ai=xlsread('ai.xlsx');
Pop=reshape(Ni,1,27);
Ai=reshape(Ai,1,27);
Ai=-Ai;
```

```
f=Pop.*Ni;
Ai=[Ai;-f];
b=[-27,0]
Times=50;
result=zeros(Times,27);
result1=zeros(5,27);
fv1=zeros(5,1);
for i=1:Times
    [result(i,:),fv(i),ex]=bintprog(f,Ai,b,[],[]);
    b=[-27,-fv(i)-i*0.0001];
    if length(find(result(i,:)~=0))==8
        result1(1,:)=result(i,:);
        fv1(1)=fv(i);
    end
    if length(find(result(i,:)~=0))==9
        result1(2,:)=result(i,:);
        fv1(2)=fv(i);
    end
    if length(find(result(i,:)~=0))==10
        result1(3,:)=result(i,:);
        fv1(3)=fv(i);
    end
    if length(find(result(i,:)~=0))==11
        result1(4,:)=result(i,:);
        fv1(4)=fv(i);
    end
    if length(find(result(i,:)~=0))>=12
        result1(5,:)=result(i,:);
        fv1(5)=fv(i);
        break;
    end
end
% xlswrite('result.xlsx',result1,1,'E1')
```

7-3 Part 3

```
%%%Question 3
Pop=xlsread('population.xlsx');
Ni=xlsread('ni2.xlsx');
Ai=xlsread('ai.xlsx');
Pop=reshape(Ni,1,27);
```

```
Ai=reshape(Ai,1,27);
Ai=-Ai;
f=Pop.*Ni;
Ai=[Ai;-f];
b=[-27,0]
Times=50;
result=zeros(Times,27);
result1=zeros(5,27);
fv1=zeros(5,1);
for i=1:Times
    [result(i,:),fv(i),ex]=bintprog(f,Ai,b,[],[]);
    b=[-27,-fv(i)-i*0.0001];
    if length(find(result(i,:)~=0))==8
        result1(1,:)=result(i,:);
        fv1(1)=fv(i);
    end
    if length(find(result(i,:)~=0))==9
        result1(2,:)=result(i,:);
        fv1(2)=fv(i);
    end
    if length(find(result(i,:)~=0))==10
        result1(3,:)=result(i,:);
        fv1(3)=fv(i);
    end
    if length(find(result(i,:)~=0))==11
        result1(4,:)=result(i,:);
        fv1(4)=fv(i);
    end
    if length(find(result(i,:)~=0))>=12
        result1(5,:)=result(i,:);
        fv1(5)=fv(i);
        break;
    end
end
```