Summary Sheet

Train stations have always been a hub for commuters, but this hub becomes a nightmare once congestion prevents people from leaving the station and getting to work. In certain train stations, the placement and size of staircases is often a limiting factor that prevents the continuous flow of movement from the train platform to the street level exit. Our models seek to minimize the time it takes to leave the train, move across the platform, and go up the stairs based on the given variables: platform length (p), number of stairs (q), train car length (d), and the number of train cars (n). Each train is at least 10 cars long, with each car having three seats on one side and two on the other. The current staircase can only hold two columns of people.

In approaching this problem, we focused on the overall time it takes to leave the station. This time was broken into three components: the time taken to unload a train and walk to the stairs, the time spent in the queue waiting to walk up the stairs, and the time needed to ascend the stairs. Our goal was to minimize the time it takes to clear the train station, meaning the time it takes the last passenger to reach street level.

We created both a mathematical and a computational model to accomplish this goal. Our mathematical model used flow rate differential equations and Euler's method estimates in an Excel spreadsheet to determine how long it takes to get through the stairway waiting queue. These time values incorporate the time to unload a train, walk to the stairs, and walk up the stairs. The computational model was a NetLogo program that modelled the time taken to completely empty a train station. For almost all of our calculations, the mathematically and computationally calculated times to empty the station were similar or equal.

Using these tools, we then modified the values of our variables to adjust the timeframe. Modifying the variables represents changing the conditions in the station platform. As we initially expected, the width of the stairwell is the most significant factor that impacts the time; allowing at least 6 people to walk up the stairs at one time halves the amount of time it takes to clear the train station. The placement of stairs, number of staircases, and even the arrangement of train cars does not have as great an influence.

To the Director of Transportation,

We understand that in your train station, the stairways are often congested during the busiest times by the amount of passengers trying to reach the exit. Thus, we propose to you a model to help improve exiting the station. In this model, we aimed to minimize the amount of time it takes to unload from a train, ascend the stairs, and exit to the street. We used a mathematical equation and a computer program to simulate this train station. Using the optimal model of a train with 600 passengers (10 cars of 12 rows), we used these tools to help minimize the time to clear out the train station. Our models suggest that the best way to improve this train station is to increase the width of the stairwell from 2 columns to 6 columns. This decreases the amount of time it takes for all passengers to leave the station by approximately one-*half*.

Adding extra stairwells, so that it triples the rate at which people leave the train platform would improve the time as well, but we believe this is not cost effective. After conducting a cost analysis, we found that widening your stairwell can save time for each unloading cycle, allowing for more train cycles per hour and thus more customers served. Overall, the increased efficiency suggests an increase in revenue of about 76-102%. Therefore, our model not only improves customer satisfaction, but it also significantly increases the amount of money earned. We highly suggest you consider adapting our model for this train station.

Sincerely,
Team 5086

P.S. As a side note, with the current amount of people the train station has crowding around the staircases, it could possibly become a hazard, especially in the case of a fire. Thus action needs to be taken to improve this train station soon.

HiMCM 2014: Problem A Solution Paper

Introduction

Background

A problem that many train stations face is congestion caused by the influx of passengers when a train unloads. Because most train stations have multiple floors, stairways often become chokepoints as passengers walk to the ground floor. In a certain central train station, train unloading is causing significant overcrowding and time-wasting. The station wishes to improve passenger exiting to optimize station efficiency. In this paper, we investigate ways to minimize the time needed to reach street level in order to exit the station. The models in this paper are relevant, with reasonable assumptions, and can applied to many different station congestion situations.

Restatement of Problem

The essential problem that needs to be answered is how the station can minimize the time needed for passengers to reach an exit at street level. **The metric that we used to measure minimal time is the time it takes for all of the train's passengers to exit the train station.** We first wish to determine a model that can minimize the time for a full train's occupants to exit and the time for two full trains' passengers to exit. However, if these results are still not optimal, we are given the opportunity to determine where the stairways can be placed.

Assumptions and Justifications

Assumption 1: There are not people standing in the car; the maximum number is filled when the seats are filled.
Justification: Since our aim to optimize the average train's unloading time, we assume that unlike a subway, there will be nobody standing up.

Assumption 2: All other conditions are normal. For example, the stairs are typical, the trains are functional and timely, people are acting orderly, and there are no extenuating circumstances.
Justification: The goal of our model is to model the everyday situation in the train station, so we will assume all functions are normal. This is also a simplifying assumption for our model.

Assumption 3: The walking speed of these passengers is 1.4 m/s.
Justification: Though there are definitely variations between people, most people, especially walking with a crowd, walk at about the same average speed of 1.4 m/s [1]. This is *w* in our model.

Assumption 4: No passengers are coming in to board the train.
Justification: This is a commuter train into the city, hence people will be exiting the train station going to work, not boarding the train to leave the city.

Assumption 5: Only one person can exit the car at a time.
Justification: Generally, the doors of a train are fit for one person, and it is of the best interest, in terms of safety and congestion, of the passengers to get off one at a time.

Assumption 6: Trains will generally pull up to the same point.
Justification: We consider it in the best interest of the station to have the trains pull up in about the same place, and most stations have dictated starting and stopping points for trains. Thus, to better represent the average train and to simplify our model, we assumed that the trains will arrive to the same spot.

Assumption 7: It does not matter what side the staircase is facing when it is not on the two edges of the station.
Justification: The difference is very small even if a staircase is facing a certain side. In addition, this helps us simplify our model by assuming both sides have the same chance to access the staircase.

Assumption 8: For multiple stairwells, the passengers will go the closest stairwell.
Justification: We assumed that passengers will choose to walk the least distance and go to the closest stairwell. We believe that this is in their best interest.

Assumption 9: The cars are completely adjoining. Thus, the door at the end of one car is in about the same position as the door in the beginning of the next car.
Justification: For simplicity, we will consider that all of the cars are joined in this way, so the distance between cars is not taken into account.

Assumption 10: Each row in the train car is 1 meter wide.
Justification: This is our approximation of the length of the seat and the legroom. This also simplifies our model by allowing us to assume that the length of the car (*d*) is the same as number of rows in the car.

Assumption 11: Train cars have the same length and the same amount of people inside. The two trains that pull up to the stations are the same length.
Justification: This is a simplifying assumption so that we do not need to take into account variance within our variables. This is also generally true for fully occupied trains belonging to the same station.

Assumption 12: The amount of time that it takes for passengers to arrange their belongings and begin leaving the train cars is 10 seconds (this is t_e in our model). The amount of time

that it takes for one row of passengers to enter the aisle in a train car is 5 seconds, one second for each person (this is t_{ar} in our model). The speed that passengers walk up stairs is 1 stair per second (this is *S* in our model) [2]. The difference between the length of the platform and the length of the train is 17 meters (this is *l* in our model). The number of steps in a staircase is 40.
Justification: These are simplifying assumptions based on what is reasonable in the real world.

Part 1: Optimizing the Single-Train Standard Layout

Given a standard train station with a platform of length *p*, and one staircase with *q* steps, our goal was to minimize the time for passengers to reach the top of the staircase to exit the complex. We began by optimizing the dimensions of the train by changing *n*, the number of cars in each train and *d*, the size of each of these cars (see Figure 1). We kept the number of passengers, given by the expression *5*n*d*, the same and changed the values of *n* and *d* to find the optimal solution. A standard train with 600 passengers was tested using (*n*,*d*) pairs of (8,15),(10,12),(12,10) and (15,8). Two complementary models were developed to describe the situation: mathematical and computational.

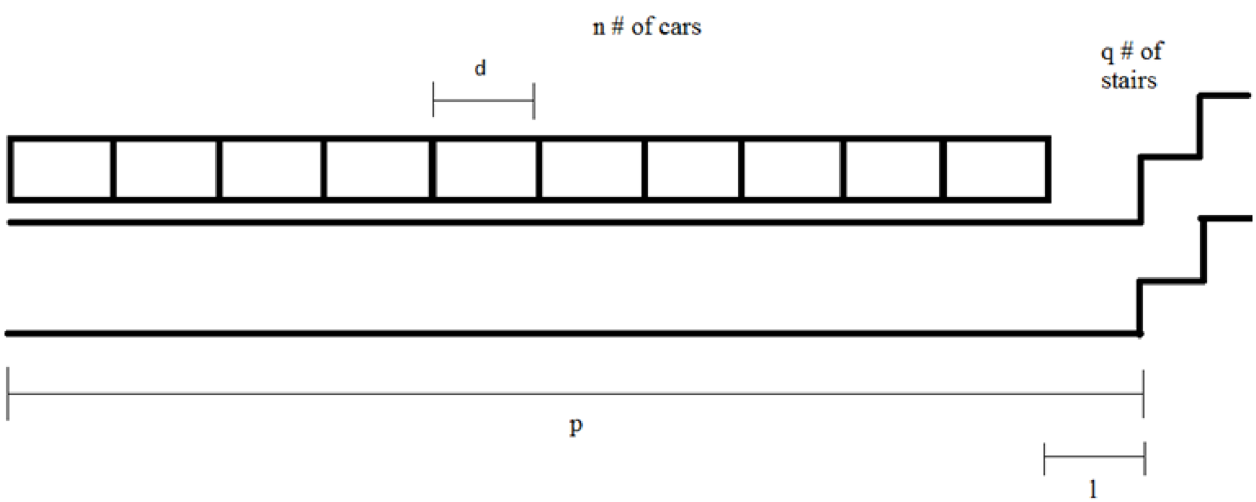


Figure 1: A Diagram of the Train Station

Mathematical Model

The efficiency of any given train setup is defined as the amount of time required for all passengers to leave the platform. This is best describes the situation because train schedules depend on the platform being as clear as possible. In order to minimize the time it takes the passengers to climb the stairs to the exit, we decided to split the time calculations into multiple steps. When the system is considered as a compartment model, any given passenger assumes one of 4 conditions at any time, *t*: (1) exiting the train car, (2) approaching the staircase (3) walking up the staircase, and (4) having left the train station.

Four variables are essential to modeling the train station: the length of the platform (*p*), the length of the car (*d*), the number of stairs (*q*), and the number of cars (*n*). We also defined the variable l, which is the length difference between the platform (p) and the length of the train (*n* * *d*). This is the extra distance passengers need to walk to reach the stairwell. First, we decided to calculate the time needed to unload a train car. The total time for the last person to unload from the car (t_c) is dependent on the time to gather one's belongings (t_e), the time for the aisle to be clear (after all rows on this passenger's side have passed through) ($t_{ar} * \frac{d}{2}$), and the time to walk across half of the train car to the nearest exit, which is dependent on walking speed (*w*) and the length of the car (*d*).

$$t_c = t_e + t_{ar} * \frac{d}{2} + \frac{d}{2w} \quad (1)$$

To find the rate of people leaving the car, we divided the number of people (5*d*), by the amount of time to empty the car after gathering one's belongings. The total time to empty the car is the difference between the time it takes the first passenger to leave and the last.

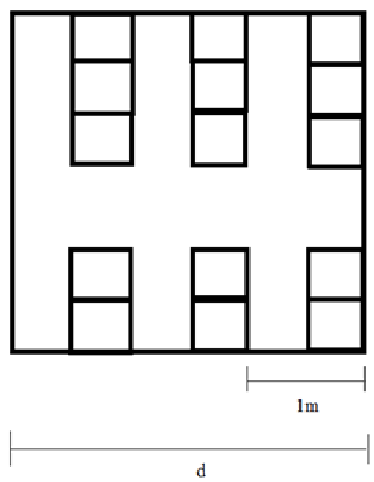


Figure 2: Diagram of a Train Car

This yields a differential equation for the rate of change in the population of the car, c. $\frac{dc}{dt}$ is negative, as the population is always decreasing.

$$\frac{dc}{dt} = -\frac{5d}{t_c - t_e} \quad (2)$$

We then substituted Equation 1 for t_c in order to determine the rate that people exited the cars. However, since there are two doors per car, the rate of people exiting a door is half the rate of exiting a car.

$$Rate\ of\ exiting\ cars = \frac{10w}{t_{ar} * w + 1}$$

$$Rate\ of\ exiting\ door = \frac{5w}{t_{ar} * w + 1}$$

We first calculated the rate of unloading through door c_1, the door closest to the stairwell. This rate is equal to the rate at which people are arriving at the stairwell. The bounds for this differential equation is when time t is greater than the time it takes for the first person to arrive at the stairwell (first person gets up, and walks to the stairwell) to the time it takes for the last person from the same train car to reach the stairwell.

$$\frac{dc_1}{dt} = -(\frac{5w}{t_{ar} * w + 1})$$

$$t \geq t_e + \frac{l}{w}$$

$$t \leq t_e + t_{ar} * \frac{d}{2} + \frac{d}{2w} + \frac{l}{w}$$

We repeated this process for door c_2, which we said was the length of the car, d, away further than door c_1 was. In addition, c_2 and c_3 based on Assumption 9, give the same equation.

$$\frac{dc_2}{dt} = -(\frac{5w}{t_{ar} * w + 1})$$

$$t \geq t_e + \frac{l + d}{w}$$

$$t \leq t_e + t_{ar} * \frac{d}{2} + \frac{d}{2w} + \frac{l + d}{w}$$

The general equation for door number N (with N=1 being the door closest to the stairwell) yielded:

$$\frac{dc_N}{dt} = -(\frac{5w}{t_{ar} * w + 1})$$

$$t \geq t_e + \frac{l + \frac{N}{2}d}{w}$$

$$t \leq t_e + t_{ar} * \frac{d}{2} + \frac{d}{2w} + \frac{l + \frac{N}{2}d}{w}$$

Once these passengers leave the first compartments, c_{1-N}, they begin their approach to the stairwell. Q represents the number of passengers in this compartment, or the queue. Using the previous equations, we calculated $\frac{dQ}{dt}$ as a function of the rate of passengers added from each train car. In addition, *Q* decreases at a rate of at most 2 people per second, because 2 people can walk up the stairs at one time, and we assumed they walk at a pace of 1 step per second. Therefore, 2 passengers leave the queue and enter the staircase per second. $\frac{dQ}{dt}$ is composed of a series of piecewise equations because the passengers' approach to the staircase is staggered based on the door from which they came. For example, for the first bit of time, door c_1 has people exiting c_1 and joining *Q* to walk up the stairs. Passengers from door c_2 and c_3 then join *Q* as well. However, at some point, door c_1 stops contributing to *Q*, since no more passengers are exiting c_1, but passengers are still coming and joining *Q* from c_2, c_3 ... c_n. The solutions of these piecewise differential equations for the (*n*,*d*) pairs (8,15) and (10,12) can be found in (Appendix A).

Excel Approximation of the Differential Equations

Because the differential equations could not easily be adapted to changing parameters, we chose to use Microsoft Excel to approximate solutions for *Q*. This Excel data spreadsheet was also programmed so that values of the numbers of cars (*n*), length of car (*d*), number of steps (*q*), stairway speed (*S*), and walking speed (*w*), the time it takes to gather belongings on a train (t_e), and the time it takes to walk in the aisle of the train (t_{ar}) could be easily modified. Below is a table of the Excel document's results for the (*n*,*d*) pairs (8,15), (10,12), (12,10) and (15,8) and the differential equation results for the (*n*,*d*) pairs (8,15) and (10,12).

Number of cars on train (*n*)	Numbers of rows of seats on the train (*d*)	Excel Approximation for the time needed for last passenger to leave (seconds)	Differential Equations Solution for the time needed for last passenger to leave (seconds)
8	15	370	368
10	12	369	374

Table 1: Excel Approximation vs Differential Equations

Because the Excel approximation was incredibly close to the real solution of the differential equations, we chose to stop solving the differential equations and use the excel approximation from this point forward.

Computational Model

To account for the fact that our mathematical equations simplify the reality of a train station, we next analyzed the situation from an agent-based perspective. By using a computational model of this train station using the modeling environment NetLogo, we can account for the variation between individual agents that is not captured by averages. The model mimics the compartment-based calculations, but also allows for a range of variation in speed and direction of individual passengers. This stochastic approach allows us to evaluate the accuracy of our deterministic mathematical model. Furthermore, this computational model allows us to easily visualize the effects of changing *w*, t_{ar}, *n*, *d*, *l*, t_e, and *q*. However due to the probabilistic nature of the computational model, there is some slight variation in the output results every time we run the NetLogo program that could not be adjusted for.

In order to create our computational model, we first considered the simplest case, which was the arrival of only one train to the station. For this case, people exit the train, moved toward one of the columns of the staircase at a speed of w, and waited to get onto the staircase, which had a maximum capacity of one person for each of the q steps. In order to complement our mathematical model, this model was used to determine the number of people in the queue and the number of people remaining in the system for every one second time interval. We iterated in one second intervals to determine the behavior of the system. The flow chart below summarizes the algorithm used to model the system (Figure 3)

Computational Model

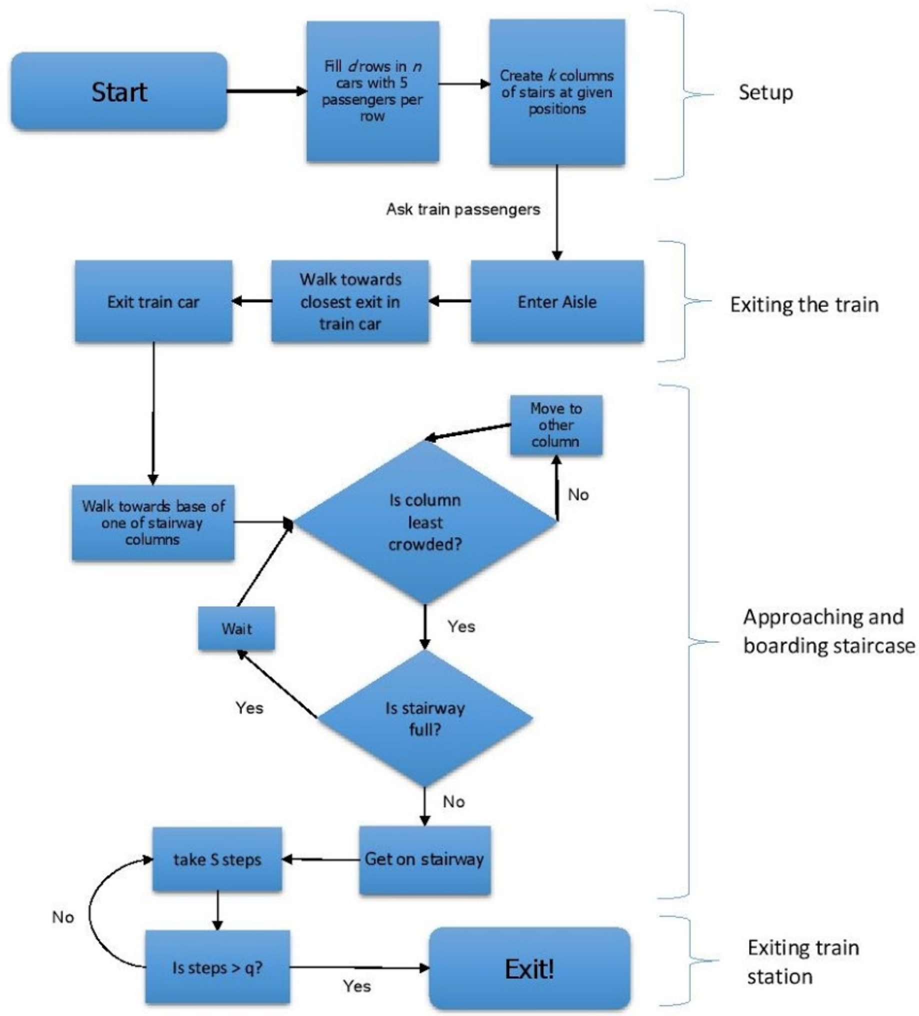


Figure 3: A flowchart of the computational model's logic

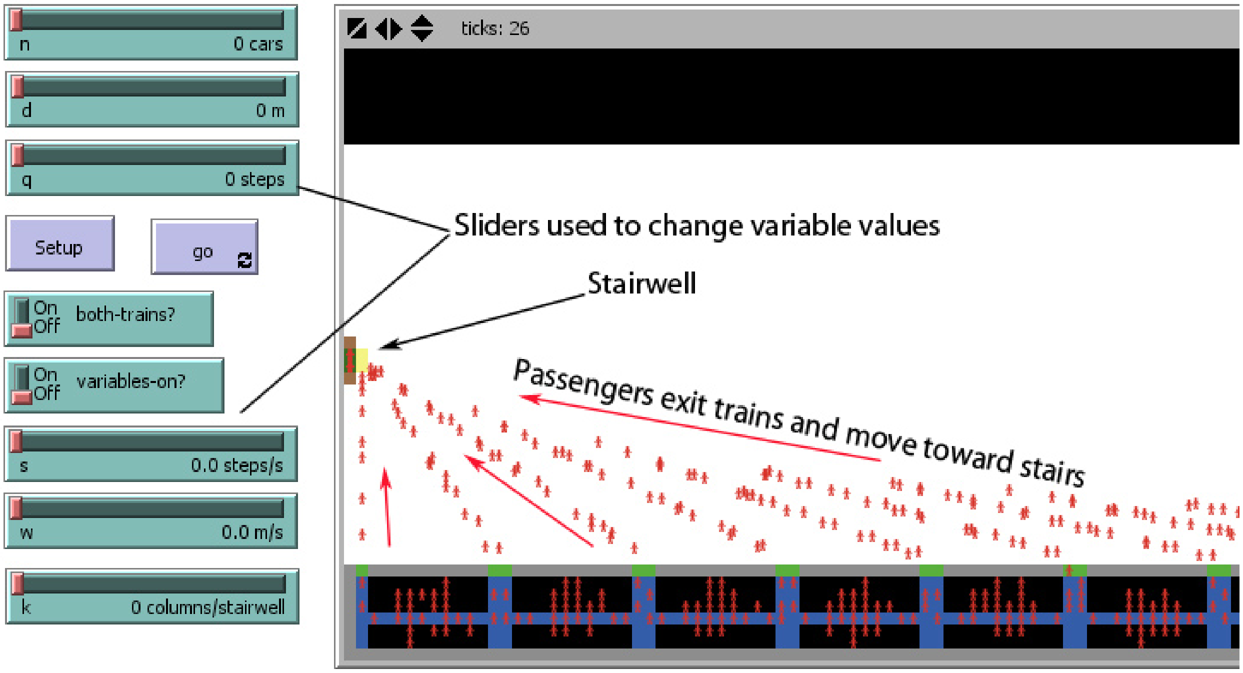


Figure 4: A Sample Screen from the Computational Model

Results

Using both our mathematical equations and our computational program, we modeled the time needed for a full train of passengers to exit the train station based upon the values of *n* and *d*. Below is a table of our results and a graph of the number of passengers over time for *n*=10 and *d*=12.

Time Needed to Clear Train Station after One Full Train Arrives

Number of cars on train (*n*)	Numbers of rows of seats on the train (*d*)	Excel Approximation for the time needed for last passenger to leave (seconds)	NetLogo data for the time needed for last passenger to leave (seconds)
8	15	370	370
10	12	369	367
12	10	368	368
15	8	368	366

Table 2: Both of the models' result times for various values of *n* and *d*

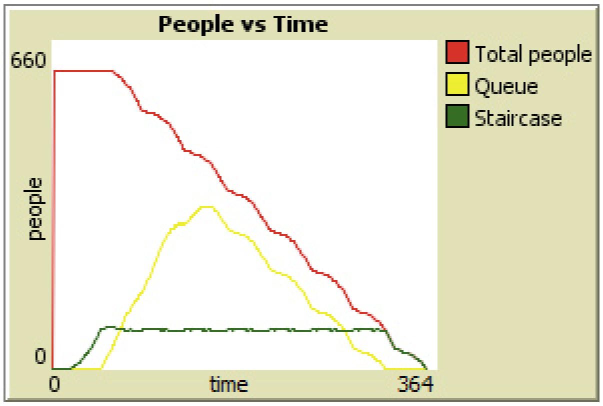


Figure 5: Number of passengers in the queue and staircase over time, collected on NetLogo

As you can see from the similar time values, the time needed to clear a train station is almost identical when comparing our differential equation estimations in Excel to the NetLogo model program. We concluded that changing the values of *n* and *d* does little to change the time needed for last passenger to leave the station. As a result, we suggest that train cars be made in the most convenient and realistic size, $n = 10$ and $d = 12$.

Part 2: Two Full Trains

The commuter train platform has tracks on both adjacent sides. The worse case scenario for the train station is if both trains, fully occupied, arrive and unload in the station at the same time. We optimized the values of *n* and *d* for this situation. To model this situation, we had to modify our mathematical and computational models slightly.

To modify the Excel approximation of the differential equations, we doubled the initial values of $c_1, c_2, c_3, etc.$ and doubled the values for $\frac{dc_1}{dt}, \frac{dc_2}{dt}, \frac{dc_3}{dt}, etc.$

The computational NetLogo model was simple to modify; we merely added a second train to the station, with both trains unloading at once. The time it takes to clear the train station when two trains arrive is significantly longer, as expected. (Table 3)

Time Needed to Clear Train Station after Two Full Trains Arrive

Number of cars on train (*n*)	Numbers of rows of seats on the train (*d*)	Excel data for the time needed for last passenger to leave (seconds)	NetLogo data for the time needed for last passenger to leave (seconds)
8	15	666	666
10	12	667	664
12	10	666	664

15	8	667	664

Table 3: Both of the models' result times for various values of *n* and *d*

But once again, it appears that a change in the values of *n* and *d* have little effect on the time it takes for the last passenger to leave the platform. Once again, we suggest that the values of *n* and *d* be set in the most practical and reasonable manner possible. We suggest that *n* be set to 10 and *d* be set to 12.

Part 3: Modifying the Staircase

Placement of the Staircase

One possible way to increase the efficiency of the train station is to place the staircase in a different location. Currently, the staircase is placed at one end of the platform, meaning that passengers in the furthest car have to walk a longer distance than passengers in the first car. We believed that this walking time discrepancy was not optimal for the setup of the staircase.

In Figure 6, we displayed possible locations of stairwells. We defined *x* as the distance of a train car *d.* Hence the staircase is currently positioned at x = 0. However, you can see that if we placed the staircase at x = 2, then the stairs would be at the end of train car 2.

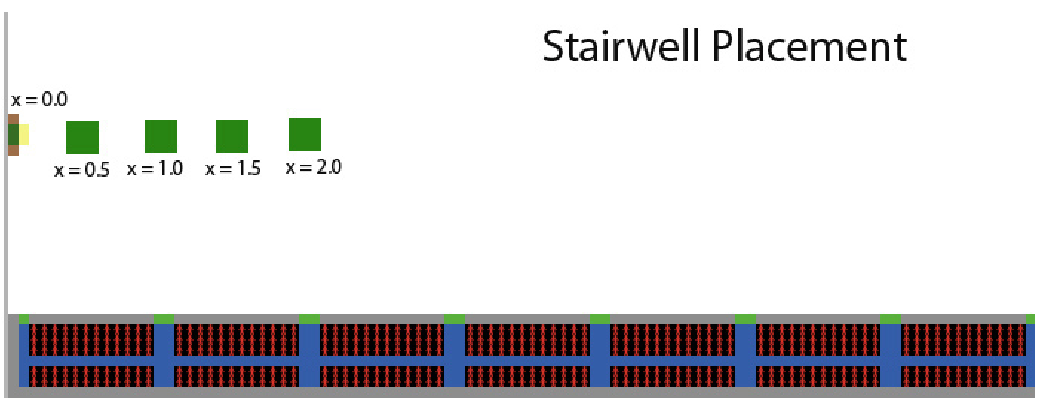


Figure 6: A Diagram explaining Stairwell Placement

We found the the time needed for last passenger to leave the platform for various values of *x* using both of our models and recorded them in the table below. We only calculated results for values of *x* between 0 and 5 because the system is symmetrical (the effect of stairs at $x = 0$ is the same as the result of $x = 10$).

Changing Staircase Placement

Location of stairwell in terms of number of train cars (x)	Excel data for the time needed for last passenger to leave (seconds)	NetLogo data for the time needed for last passenger to leave (seconds)
0	356	367
0.5	355	366
1	352	368
1.5	355	366
2	352	365
2.5	355	366
3	352	364
3.5	355	366
4	352	362
4.5	355	366
5	352	362

Table 4: The times needed to clear the train station based on n = 10, and d = 12 and various values of x

There is a clear pattern in the data. According to both models, the best position for the staircase occurs when x equals either 4 or 5. Conceptually, this makes sense because the staircase is in the middle of the train and reduces the maximum distance that a passenger has to walk. Unfortunately, this value of x = 4 or 5 is not universal, because a train can vary in length; n could equal 10 cars, or n could equal 20 cars. Therefore, we cannot make a sweeping generalization that the stairwell placement of x equals 4 or 5 is optimal; we can only say that the best place to put a staircase would be halfway down the length of a train.

Part 4: Adding more Stairwells

After determining that changing the location of the stairwell is not useful because it depends on the length of the train, we wanted to see if adding multiple stairwells would be useful as well. This portion of our model was conducted entirely in Excel due to limitations in our NetLogo computational program. The Excel program was modified under the assumption that people will go toward the stairwell closest to them.

We tested whether implementing 2 or even 3 stairwells improves the time it takes for the train station to clear. Once again, placement of our stairwells were dependent on *x*, which was relative to the length of a train car *d.*

First we placed two and three stairwells side-by-side at the end of the platform near the front car of the train, this is analogous to doubling and tripling the width of the first stairwell. We found that the time it takes is 223 seconds and 183 seconds respectively to clear the train station. This is equal to the time it takes if you were to widen one stairwell from 2 columns to 4 or 6 columns (shown in Part 5).

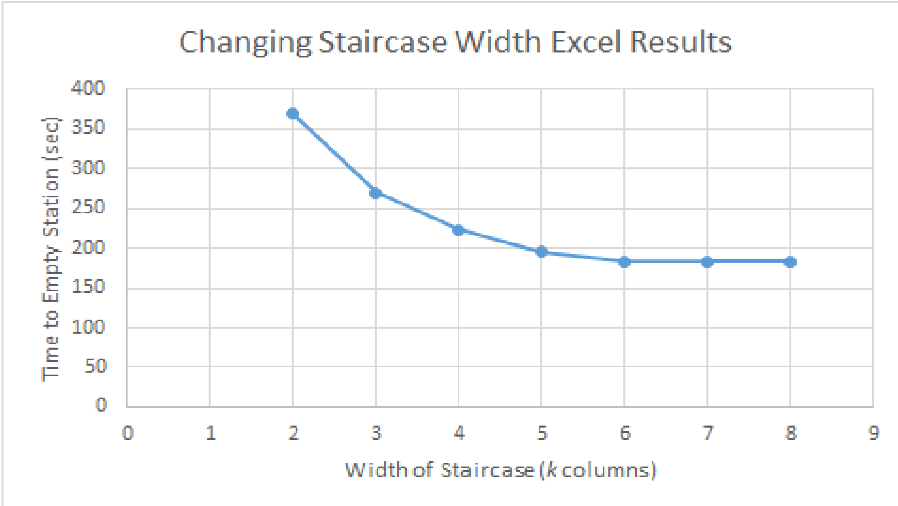
We then placed stairwells in different locations. The location of 2 stairwells that minimizes the time needed to empty the train station is one at the very front of the platform, and one 10*x* away. (This time value is 221 seconds). Any other arrangement of two staircases was no shorter than 223 seconds, meaning that you might as well widen the staircase rather than add more staircases. And once again, this placement varies with the length of a train car. So if a train is longer than *n* = 10, then the stairs would need to be in a different location, which is impossible after stairs have already been built. The location of 3 stairwells that minimizes the time is actually when all three are at the end of the platform near the front car of the train. All other arrangements have an average clearing time of 250 seconds. In light of this, we further analyze simply adjusting the parameter *k*, the number of columns in each staircase.

Part 5: Changing the Width of the Staircase

From our NetLogo program, differential equations, and from common sense, you can see that one the main problems with this train station is that people crowd around the staircase, trying to exit the platform. However, only 2 people can go up the staircase at once, meaning that a queue builds up where people wait to walk up the stairs. Time is lost just waiting for the stairs. Hence one way to solve this problem is to widen the staircase and allow more people to walk up at once. If *k* people can walk up the staircase at once, then the width of the staircase is also defined as *k*.

By modifying the width of our stairwell in NetLogo, we can easily account for this change. Modifying our Excel model was equally simple. In the original $\frac{dQ}{dt}$, *Q* decreases at a rate of 2*S* because there are only 2 columns of people who can walk up the stairs at once. But with an increasing width *k, Q* decreases at a maximum rate of *ks.*

We simplified this issue and assumed that only one train arrives at a time. We tested different *k* values in both our Excel spreadsheet and our computational model, with the set parameters of walking speed (*w*) = 1.4 m/s, stair-climbing speed (*s*) = 1 m/s, the number of cars (*n*) = 10, the length of the car (*d*) = 12, and the number of steps on the stairs (*q*) = 40. In both models, we found that after *k = 6*, the time it takes to clear the train station does not change. At *k* = 6, the estimated time is 183 seconds in Excel and 209 seconds in NetLogo. Yet until k = 6, there is a significant decrease in time for every increase in k (Figure 7).



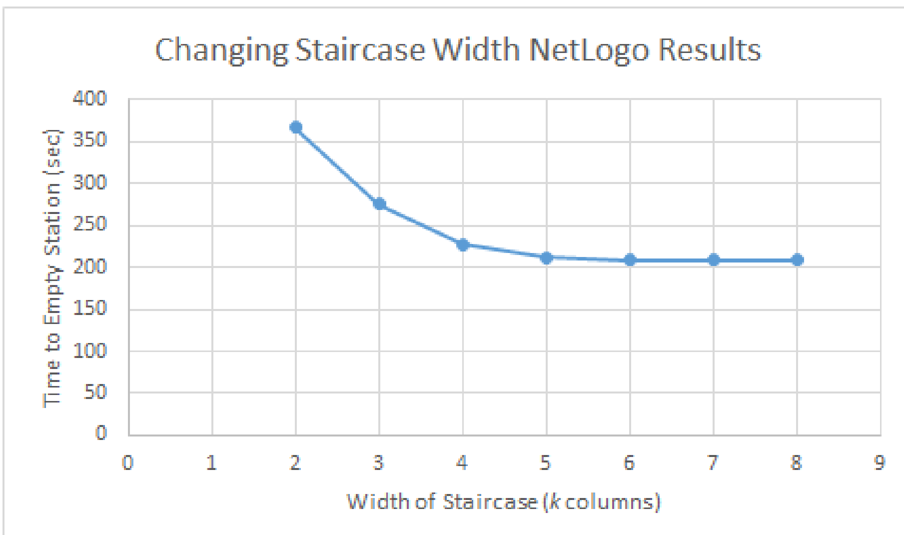


Figure 7: These are the times needed to clear the train station based on n = 10, and d = 12. The k value is the new width of the staircase. The number of columns of people that can walk up the staircase is equal to the width k.

From the above graphs we can conclude that, according to both models, the minimum time to empty the platform occurs at values of $k = 6, 7 \text{ } and \text{ } 8$. To differentiate between these values, we must look at additional factors that influence the train station designer's decision such as the cost of additional columns and the revenue lost from high wait times. While every column carries construction costs, it also increases the number of train cycles that can be scheduled per hour because passengers are exiting more quickly. Using an average ticket cost of approximately $30.00 [3] and cost per column added of $4,000.00 [4]. From this data, we created the following cost benefit analysis chart.

k	One Time Cost	1-Year Percent Change in Revenue Excel	1-Year Percent Change in Revenue NetLogo
2	$0.00	0.00%	0.00%
3	$4,000.00	36.67%	32.97%
4	$8,000.00	65.47%	60.96%
5	$12,000.00	89.23%	73.11%
6	$16,000.00	101.64%	75.60%
7	$20,000.00	101.64%	75.59%
8	$24,000.00	101.63%	75.59%

Table 5: This table compares the 1-Year Percent Change in Revenue for both models and various values of *k*

From this table, we can conclude that the most cost effective value of *k* is 6 columns. Thus, we suggest that 4 stairwells be added to reach maximum cost-effectiveness.

Other Extensions

Escalators

Another way that the train stations can reduce the blockage in front of the steps is with the implementation of escalators. Because escalators move faster than the average person can, the escalator can move people significantly faster. To model this we changed our value for *S*, the speed at which passengers climb stairs per second to account for the speed of the escalator (S=1.3 m/s) [5]. The results are depicted in the table below.

Escalators?	Excel data for the time needed for last passenger to leave (seconds)	NetLogo data for the time needed for last passenger to leave (seconds)
No	369	367
Yes	199	192

Table 6: Escalators vs Stairs data for both models using $n = 10,\ d = 12$, and $k = 2$

From this table, we can conclude that an escalator is significantly faster than the stairs. We suggest that the stairs should be replaced with escalators.

Sensitivity Analysis

To test how sensitive our results are to changes in the inputs, we can change our independent variables and see how much the resulting time is affected. We changed one variable by a certain percent with the rest remaining constant. Below is a chart showing tests for *n*, *d* and *S*.

Independent Variable and Percent Change	Percent Change in the Resulting Time Excel	Percent Change in the Resulting Time Netlogo
+75% *n*	+62.2%	+56.25%
+75% *d*	+61.5%	+61.6%
+80% *S*	-46%	-47.7%

Table 7: This is Sensitivity Analysis for the independent variables *n*, *d*, and *S*

The results show that the system is sensitive to changes in *n,* and *d*, and less sensitive to *S*. Furthermore, all of our results suggest that there is little variation between our compartment-based and agent-based models. Thus, we conclude that our model is *not* sensitive to the virtually random variations in walking speed and direction for passengers.

Strengths and Weaknesses

Strengths

Our model is easily adaptable to changes in different variables: Our model can represent many different situations by simply changing the variables, such as walking speed, length of platform, stair-climbing speed. Thus, it is easily applicable.

The two parts of our model confirm each other: Both our computational and mathematical models give similar values, which allows relative confidence in the accuracy of both of our models.

Our model can account for some randomness: Due to the computational model's usage of the random movement of agents along with given values, our model is relatively realistic. In addition, randomness can be easily programmed into our NetLogo model, allowing us to more accurately represent a real world situation. Specifically, our NetLogo model incorporates individuals' variation from the average walking speed and accounts for the decrease in speed for passengers when walking in a crowd.

Weaknesses

Our model runs slower with high amounts of passengers: This is especially true for our computational model. High numbers of agents in our computational model slows the program down considerably. In addition, the excel sheet used to approximate our values from our differential equation becomes very lengthy with large numbers of passengers. This impedes our ability to run large numbers of trials for more robust analysis.

Our model lacks robustness to extreme situations: For example, if there were to be an extremely slow or extremely fast person that disrupts the flow of traffic, our model would not be able to take that into account. In addition, our model would not take extenuating circumstances, like stair breakdown, into account well.

Our model is based on estimated values: Our values are based on the Excel approximation of the solutions of our differential equations. Though the approximations were relatively accurate when we compared a few to the actual solutions of the differential equations, they are still not completely accurate.

Our NetLogo model is unable to increase the number of stairwells: Our NetLogo program cannot account for increased number of stairways due to programming limitations. This means that we must rely on the Excel spreadsheet and differential equations mathematical model to test the timeframe needed to clear the train station.

Conclusion

Using both of our models, we determined and minimized the time it took for the last person to exit the station. We determined that with our set capacity of 600 passengers, the best train dimensions to have was a train with 10 cars each with a length of 12 meters, in our case 12 rows. This is true for both one and two full trains pulling up to the station. In addition, we found that, if staircases are movable and the train length is known, the optimal place for the staircase is in the center. However, train lengths vary, so the optimal staircase place will also vary. Hence it is simpler to leave the staircase where it is at the head of the platform near the first train car.

We determined that based on our model and realistic assumptions, it would be best to increase the width of the staircase instead of adding new staircases. Though adding new staircases causes about a few seconds decrease compared to increasing staircase width, it is not worth the extra cost. Thus, an increase in the width, specifically to 6 columns, would greatly improve traffic flow. We also chose to extend our model to incorporate escalators, determining that they were definitely beneficial to have in this system. Doing these adjustments and comparisons have demonstrated that our model is strong and easily adjustable for many important variables.

We would improve our model by using actual rather than estimated values for the solutions of our differential equations, increasing our accuracy. We would also further our sensitivity analysis, testing the effect of other variables, like p and q, on our model. Since we chose to minimize the overall exit time, minimizing each individual's exit time and the standard deviation between the passengers would improve our model too.

Furthermore, we would like, with more time, to examine other methods that the station could use to decrease congestion after the unloading of a crowded train. Specifically, we would look into model that would give a more random movement of the people, since in reality people vary in their walking speed and path direction. Another approach to the problem we would like to investigate is how door size affects the time. We want to see how allowing more people to fit through a door at once will affect our optimization.

References/Citations

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Appendix

A:

$$\frac{dQ}{dt} = \begin{cases} 0 \text{ if } 0 \leq t < \frac{155}{7} \\ 0 \text{ if } \frac{155}{7} \leq t < \frac{230}{7} \\ \frac{5}{8} \text{ if } \frac{230}{7} \leq t < \frac{305}{7} \\ \frac{19}{8} \text{ if } \frac{305}{7} \leq t < \frac{380}{7} \\ \frac{33}{8} \text{ if } \frac{380}{7} \leq t < 65 \\ 5 \text{ if } 65 \leq t < \frac{755}{7} \\ \frac{33}{8} \text{ if } \frac{755}{7} \leq t < \frac{830}{7} \\ \frac{19}{8} \text{ if } \frac{830}{7} \leq t < \frac{905}{7} \\ \frac{5}{8} \text{ if } \frac{905}{7} \leq t < 140 \\ -\frac{9}{8} \text{ if } 140 \leq t < \frac{1055}{7} \\ -2 \text{ if } \frac{1055}{7} \leq t \end{cases}$$

Appendix Equation 1: Differential equation for (*n*,*d*) pair (8,15)

$$Q = \begin{cases} 0 \text{ if } 0 \leq t < \frac{155}{7} \\ 0 \text{ if } \frac{155}{7} \leq t < \frac{230}{7} \\ \frac{5}{8}(t - \frac{230}{7}) \text{ if } \frac{230}{7} \leq t < \frac{305}{7} \\ \frac{19}{8}(t - \frac{305}{7}) + \frac{375}{56} \text{ if } \frac{305}{7} \leq t < \frac{380}{7} \\ \frac{33}{8}(t - \frac{380}{7}) + \frac{225}{7} \text{ if } \frac{380}{7} \leq t < 65 \\ 5(t - 65) + \frac{4275}{56} \text{ if } 65 \leq t < \frac{755}{7} \\ \frac{33}{8}(t - \frac{755}{7}) + \frac{2355}{8} \text{ if } \frac{755}{7} \leq t < \frac{830}{7} \\ \frac{19}{8}(t - \frac{830}{7}) + \frac{9375}{28} \text{ if } \frac{830}{7} \leq t < \frac{905}{7} \\ \frac{5}{8}(t - \frac{905}{7}) + \frac{20175}{56} \text{ if } \frac{905}{7} \leq t < 140 \\ -\frac{9}{8}(t - 140) + \frac{10275}{28} \text{ if } 140 \leq t < \frac{1055}{7} \\ -2(t - \frac{1055}{7}) + \frac{19875}{56} \text{ if } \frac{1055}{7} \leq t \end{cases}$$

Appendix Equation 2: Solution to Appendix Equation 1

$$Q = \begin{cases} 0 \text{ if } 0 \leq t < \frac{155}{7} \\ 0 \text{ if } \frac{155}{7} \leq t < \frac{215}{7} \\ \frac{5}{8}(t - \frac{215}{7}) \text{ if } \frac{215}{7} \leq t < \frac{275}{7} \\ \frac{19}{8}(t - \frac{275}{7}) + \frac{75}{14} \text{ if } \frac{275}{7} \leq t < \frac{335}{7} \\ \frac{33}{8}(t - \frac{335}{7}) + \frac{180}{7} \text{ if } \frac{335}{7} \leq t < \frac{395}{7} \\ 5(t - \frac{395}{7}) + \frac{855}{14} \text{ if } \frac{395}{7} \leq t < \frac{755}{7} \\ \frac{33}{8}(t - \frac{755}{7}) + \frac{4455}{14} \text{ if } \frac{755}{7} \leq t < \frac{815}{7} \\ \frac{19}{8}(t - \frac{815}{7}) + \frac{2475}{7} \text{ if } \frac{815}{7} \leq t < 125 \\ \frac{5}{8}(t - 125) + \frac{5235}{14} \text{ if } 125 \leq t < \frac{985}{7} \\ -\frac{9}{8}(t - \frac{985}{7}) + \frac{2655}{7} \text{ if } \frac{985}{7} \leq t < \frac{1045}{7} \\ -2(t - \frac{1045}{7}) + \frac{5175}{14} \text{ if } \frac{1045}{7} \leq t \end{cases}$$

Appendix Equation 3: Solution to the differential equation for (*n*,*d*) pair (10,12)