Director of Transportation,

We're attempting to optimize the configuration of staircases for a platform. In doing this, the most important thing to consider is the passenger's safety and happiness while exiting. In our model we strive to minimize first the amount of time it takes for a passenger to exit a train and get to street level and then the amount of time that a passenger stands waiting to use the staircase. We did this based on the assumption that people would be happiest and safest if they can exit the platform quickly without standing in a mob in front of the opening of the stairs.

Keeping this in mind, we compared different platform configurations. For example, although putting one staircase at one end of the platform and putting one staircase in the middle of the platform yields effectively the same time for each passenger to get to street level, in the first configuration the time people arrive at the opening of the stairs is more staggered and therefore people are waiting for the stair for less long. This makes the first configuration better.

Of course, adding more staircases reduces both the total and waiting time for each passenger. Of the configurations with two separate staircases, the one that minimizes these times the best has the stairs on either end of the platform. Anything with the stairs closer to the center of the platform increases waiting time, and a configuration with the two staircases together increases the total time.

We find that model allows us to configure a platform to enhance the process of exiting the station for passengers in the most optimum way. Adopting our model would improve overall passenger safety and happiness.

Thank you,
Team 5089

Summary Sheet

Our problem deals with configuring the platform of a commuter train station in a way that minimizes the amount of time that it takes for passengers to exit the train and climb the stairs to street level. We used a worse case scenario of a full train of ten cars arriving at the platform and unloading all of its passengers, who then go directly to the staircase to exit the station. We optimized our configuration by looking at minimizing the average time spent getting to street level, and then minimizing the time spent waiting in line to get to the staircase.

To solve this problem, we divided exiting the platform into three sections of time: time to get off the train and to the staircase, time spent waiting at the stairs, and time spent climbing the stairs. We first found walking speed to be normally distributed. Then, using both a numerical and analytical approach, found the distribution of arrival times of individuals to the staircase(s). The numerical approach used a program to procedurally generate scenarios and average them for an accurate result. The analytical approach found the cumulative distribution function associated with aggregate inverse scaled normal distributions. Once this data or distribution had been acquired, by taking data on how long it takes the average person to walk up a set of stairs, it was then possible to find both the average time to get to a staircase and the average time spent waiting. The time to ascend the stairs was unrelated to this distribution and found separately to be fixed. Because we assumed that spending time walking to the staircase was preferable to waiting en masse in front of the stairs, we found that positioning staircases at the ends of the platform was more optimal than having them towards the middle.

Both models recommend three configurations that each work best in a different way. If it's a priority to minimize the overall time, a configuration with three staircases positioned $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{5}{6}$ of the way along the platform. However, if it's a priority to minimize the crowding, two staircases at either end of the platform is optimal. Finally, if it is desirable to have only one staircase, it is best to position it on one end of the platform.

HiMCM Problem A: Unloading Commuter Trains

Team 5089

November 2014

1 Introduction

Commuter train platforms are generally some of the busiest places in a city. Thousands of people enter and exit every day, resulting in massive delays and crowds around the exits. People are entering and leaving the station, transferring trains, or simply staying in the same one. With all this in mind, it may be difficult and time-consuming for each passenger to leave the station. There is often a "fan" of passengers near the stairway, as it cannot accommodate a large number of people all at once. In the busiest scenario, a train entirely full of commuters arrives at a platform, and every passenger gets off to exit the station. Though the platform will crowd up for just one train full of passengers, the crowding would be even worse if two full trains happen to arrive at the same time.

In this problem, we look at a general platform and train. Given certain properties of the station and the train, we attempt to develop a model that measures the time it takes for a commuter to reach street level and exit a station and determines an optimum platform configuration, through placement of staircases. We also consider how the time would be affected if there is a second train that arrives at the same time. Lastly, we looked at seeing how the amount of people the staircase can accommodate would change the time, as well as how the number of stairways would affect it.

2 Given Information

- Each train consists of at least 10 cars and is full to capacity. Each car has 2 exits, one near each end. There is a center aisle with two seats on one side and three on the other, for each row of seats.
- After exiting the car, all passengers must walk to the stairway to exit the station at street level.
- The stairways can only accommodate two columns of people exiting to the top of the stairs.
- Each row of seating has an aisle with two seats on one side and three on the other.

3 Assumptions and Justifications

- **Each train car's doors split the car into equal parts (thirds).** This is based on the R143 car, used by the New York City Subway. Based on other references, the dimensions of this car seems to be representative of the average car size. (Reference 1)

- **There is a three foot gap between each row of seats.** Each row of seats has to have a gap in order to allow room for people to be seated. Most seating arrangements on public transportation vehicles seemed to have roughly this amount of space, so we decided to use it as well.
- **All staircases are the same dimensions, with a height of two stories (6 meters) and where each step is 6 inches (.15 meters) tall.** This was based on average staircase dimensions and standard building codes. The width of the platform was assumed to be 50 feet, and the staircases were placed exactly in between the two rail tracks, so 25 feet from each.
- **Walking speeds follow the normal distribution $N(1.34, 0.37)$.** We assumed it was 1.5 m/s, or 4.92 feet/sec. From this reference, we also assumed the average speed for climbing up stairs was .442 m/s. We reached a final value of 4 people/sec, which says the stairway lets 4 people exit every second. Based on the problem, let the number stairs in each staircase be q. (Reference 3)
- **The same number of people leave each car door.** We assume that for each car, half of the occupants go to each door to minimize the wait time to get out of the train. People would also want to leave the car as soon as possible, so they would go towards the exit with fewer people; because the doors are evenly spread apart on the car, half of the car should exit through each.
- **There is nobody standing in the train.** The problem statement gives us the seating arrangement for each row, so we assumed the entire occupancy comes from people seated in the train, and that there is no one standing in the aisle. Additionally, with the dimensions of the train car, it would be relatively unreasonable for people to stand in the aisle.
- **There are no more than three staircases being added.** Based on the size of our platform and the number of people who could be on the platform, we found it would be unreasonable to have more than three staircases.
- **The stairway is at the end of platform.** The initial configuration is to have one stairway at one end of the platform. This is the configuration in many commuter stations, such as the Center City Commuter Connection at the Market East Station in Philadelphia.

4 Models

4.1 Basic Quantities

Both the analytical and numerical models require the definition of some fundamental variables. They are as follows:

$$n = \text{number of cars to a train}$$

$$d = \text{length of each car}$$

$$p = \text{platform length}$$

$$h_c = \text{horizontal distance between gate i and the stairs}$$

$$q = \text{number of stairs in each staircase}$$

$$\beta = \text{number of stairwells}$$

There is a 3 foot difference between each row, and 5 seats per row, which gave us the above equation, so the number of people per car can be represented by: $\frac{5d}{3}$, making the capacity, or total number of people in the station:

$$C = \frac{5nd}{3}$$

where C is the total number of people in the train station. So when $d = 60$ and $n = 10$, the train can hold 1000 commuters.

Given this information, it is possible to use either model in order to perform projections and determine the amount of time it takes the average passenger to leave the station. The diagram below gives the general diagram that assigns values to a few of the above variables. Each dot represents a door and the number above is its assigned door number. In this case, the length of each car is 60 feet, and length of the platform is 50 feet. These values came from averages that we found online. We also made n equal to 10. From our assumptions, the doors split the car into equal parts, so the doors on each car are 20 feet apart. The drawing also depicts how simple geometry was used to calculate the walking distance to the staircase. This process was repeated for every door on the train. The number of doors on the train is simply $2n$, because there are two per car.

First, we would need to find the walking distance for a passenger from a certain door. Let c be

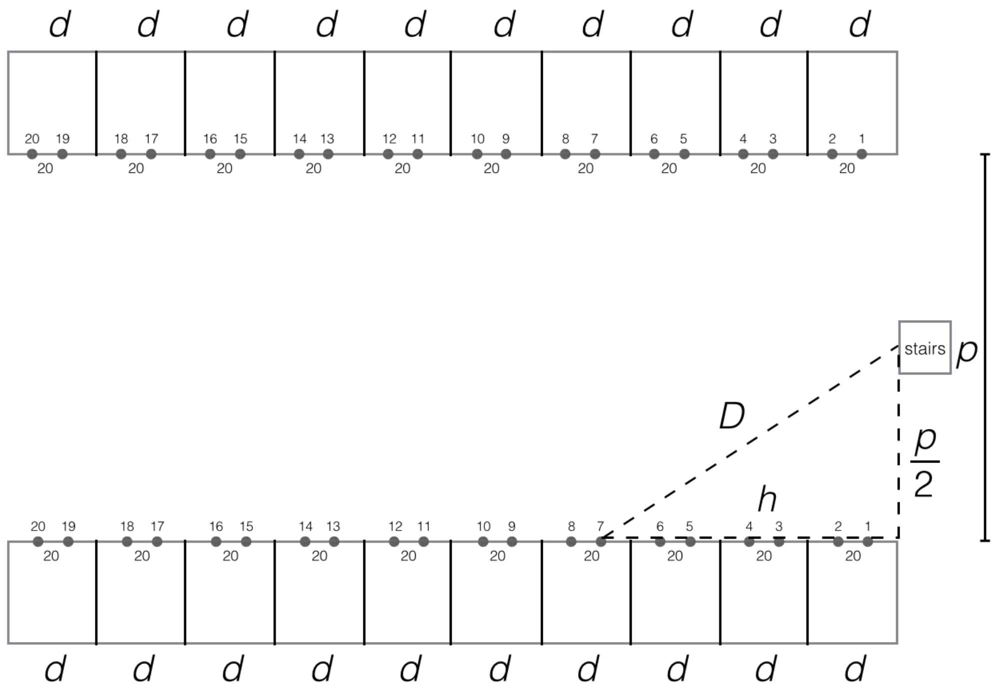

Figure 1: Diagram of central station

the door number and let D be the walking distance for a passenger from a certain door. D can be represented as:

$$D = \sqrt{\left(\frac{p}{2}\right)^2 + (h_c)^2}$$

where h is defined as:

$$h = \begin{cases} \text{60c-40} & \text{if c mod } 2 \equiv 1 \\ \text{60c-20} & \text{if c mod } 2 \equiv 0 \end{cases} ; c \in \{1, 2, \ldots, 2n\}$$

Then, we looked at the time it took for a person to reach the end of a staircase, or the rate at which people move on/off the stairway. We obtained a value of 0.15m for the height of a stair, and from 0.442m/s as the average climbing rate, we calculated a value of about 3 footsteps/sec. This is about 1 footstep for every $\frac{1}{3}$ of a second. Because it takes two footsteps to get up one step, this translates to 1.5 people per step and then to 3 people per full step. We decided that because people would be crowded up the stairs at more than one person every other step, we should introduce a buffer that accommodates for this, which gives us 4 people per second as the speed of traveling up the staircase.

4.2 Numerical Model

In order to calculate time spent to get to ground level, we separate the different time periods: time spent for each person walking to the stairway, T_i, the time spent waiting to get onto the stairs, W, and the time spent walking up the stairs, S. This can be presented by the equation below.

$$T_{\text{total}} = \overline{T} + W + S$$

The time each person spent walking to the stairway, T_i, is calculated by the equation below.

$$T_i = \frac{D}{N(4.396, 1.213)}$$

The average of those T_i values is then calculated for 100 runs with a computer program (See appendix). Essentially, $\overline{T}$ is determined by finding the average walking times for 100 times the capacity of the train(s) in the station.

To calculate average wait times, W, we find the number of people who have already made it to the street by subtracting the time it takes for the first person to get to the stairs from the time it takes for the average person to get to the stairs, which we then multiplied by 2Q to get the number of people who have left when the last person gets in line.(Because arrival times are fairly linear, we can approximate the arrival time of the final person by doubling the arrival time of the average person.) We subtract this from the capacity, C, to get the number of people left in line. Dividing this quantity by 2Qβ gives the wait time for the average person.

$$W = \frac{C - 2\left(\overline{T} - T_{\text{Min}}\right)Q}{2Q\beta}$$

Using empirical data, we found that it takes about 25 seconds to climb up 40 stairs. We used this as a proportion for the rate at which it takes one to climb up stairs, and ultimately determined S with the equation below.

$$S = \frac{25}{40} \times q = \frac{5q}{8}$$

4.2.1 Example Model

For our initial train car setup, we assumed 10 cars per train, so that the entire train length was 600 feet. We assumed of the length of the platform, p, was 50 feet. The doors on each car were placed 20 feet from each end, resulting the doors on each car being 20 feet apart from each other. Inside each car there is a side with two seats and a side with three seats, separated by an aisle. With our assumption that each row takes up three feet of space there are a total of 20 rows in each car. This was the basic setup of every car, as shown in Figure 1.

Initially, the configuration with only one staircase was used. Other configurations were tested, as explained later in the paper. The staircase was placed at the end of the platform (the end of the train) in the between both tracks. This gives the staircase a distance of 25 feet from each rail track. For each door on a car, we calculated the distance to the staircase, and the time it would take for each group of people from that door to reach the staircase. The following calculations shows how we reached the number of passengers per door.

Based on our assumption, 50 people exit from each door, because $100/2 = 50$.
Next, we used the Pythagorean Theorem to find the distances to the stairway from each door. The time to reach the stairway was simply done by dividing the distance by the average walking speed, 4.92 feet/sec, as stated in Section 4. $q = 6/.15 = 40$ steps

Horizontal Distance of Gate from Staircase (feet)	Passenger's Walking Distance to Staircase (feet)	Time To Reach Staircase (sec)
20	32.0	6.5
40	47.2	9.6
80	83.8	17.1
100	103.1	21.0
140	142.2	28.9
160	161.9	32.9
200	201.6	41.0
220	221.4	45
260	261.2	53.1
280	281.1	57.1
320	321.0	65.2
340	340.9	69.3
380	380.8	77.4
400	400.8	81.5
440	440.7	89.6
460	460.7	93.6
500	500.6	101.8
520	520.6	105.8
560	560.6	113.9
580	580.3	118.0

Table 1: Table depicting the distance and time from each exit on the train

After analyzing the data from this table and based on our assumption that the staircase is two stories high, we found that $q = 40$. Based on the average speed for climbing up the stairs, we determined it would take 25 seconds for each person to go up all of the stairs.

4.2.2 General Method

In the method above, we worked under assumptions given by the problem, but this method can be made to be made completely abstract: able to work with any platform configuration.

Adding Multiple Stairways

The method above assumed only one stairway will be used and it will be placed at an end; however, multiple configurations can be built if we change the position of that stairway, as well as adding additional stairways. This is done by allowing the initial stairway to lie at the point $\left(0, \frac{p}{2}\right)$ and allowing other stairways to have x coordinates of up to nd. This would change our calculation of D to be:

$$D = \sqrt{\left(\frac{p}{2}\right)^2 + (h_c - \beta_{\text{Min}})^2}$$

where β_{Min} is the closest stairwell.

Varying Car Number and Size

In the method above we assumed d and n were constants that were later used to get times; however, these constants can be changed. Variability of these constants force h to be redefined as:

$$h = \begin{cases} \text{d}/3 \times \left(\frac{3c}{2} + 2\right) & \text{if c mod } 2 \equiv 1 \\ \text{d}/3 \times \left(\frac{3c}{2} + 1\right) & \text{if c mod } 2 \equiv 0 \end{cases} ; c \in \{1, 2, \ldots, 2n\}$$

Allowing A Second Train

In order to account for the need to accommodate multiple trains on the platform, the model must simply be changed to double the carrying capacity of a single car, assuming the additional train is across the platform, of the same configuration (n and d values are equivalent), and opposing car doors will open at the same time. Under the constraints of our model, all optimal configurations developed for a single train is also viable for a two train system.

4.3 Analytical Model

While the numerical model produced accurate results on a whole, we noticed that there was a tendency for large outliers to appear in the data. For instance, over individual runs, sometimes the combination of a slow walker and the longest walking distance would produce one or two individuals who would take over ten minutes to reach the stairway. In order to address the problem in an alternative way, we created the following analytical model.

We begin the analytical model with the given assumption that walking speeds follow the normal distribution $\omega \sim N(1.34, 0.37)$. We would like to find the distribution of the arrival times of individuals at the stairs, Y, which has the relation $Y = \frac{D}{\omega}$. If F_Y is the cumulative distribution function of the random variable Y and F_ω is the cumulative distribution function of the random variable ω, then we can do the following to establish a relation:

$$\begin{aligned}
F_Y(y) &= Pr(Y \leq x) \\
&= Pr(\omega^{-1} * D \leq x) \\
&= Pr\left(\omega^{-1} \leq \frac{x}{D}\right) \\
&= Pr\left(\omega \geq \frac{D}{x}\right) \\
&= 1 - Pr\left(\omega < \frac{D}{x}\right) \\
&= 1 - F_\omega\left(\frac{D}{x}\right)
\end{aligned}$$

Because we know that the probability density function of a random variable is the derivate of its cumulative distribution function, we also know:

$$\begin{aligned}
F_Y(y) &= 1 - F_\omega\left(\frac{D}{x}\right) \\
f_Y(y) &= \frac{D}{y^2} f_\omega\left(\frac{D}{x}\right)
\end{aligned}$$

We know the probability density function of ω because it follows a normal distribution, so:

$$\begin{aligned}
f_\omega(x) &= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\omega^2}} \\
f_Y(x) &= \frac{D}{y^2} f_\omega\left(\frac{D}{y}\right) \\
&= \frac{D}{y^2\sigma\sqrt{2\pi}} e^{\frac{-\left(\frac{D}{y}-\mu\right)^2}{2\sigma^2}}
\end{aligned}$$

Because D was defined earlier to be $\sqrt{(h_c)^2 + \left(\frac{p}{2}\right)^2}$, we can substitute in D to make $f_Y(x)$ a parametric equation.

$$f_Y(x) = \frac{D}{y^2\sigma\sqrt{2\pi}} e^{\frac{-\left(\frac{D}{y}-\mu\right)^2}{2\sigma^2}}$$

$$f_Y(x; h_c, p) = \frac{\sqrt{(h_c)^2 + \left(\frac{p}{2}\right)^2}}{y^2\sigma\sqrt{2\pi}} e^{\frac{-\left(\frac{\sqrt{(h_c)^2+\left(\frac{p}{2}\right)^2}}{y}-\mu\right)^2}{2\sigma^2}}$$

For a given train with n cars and doors at locations h_c, we can then sum up and average the probability density functions to get the aggregate probability density function of the entire population of the train.

$$f_{total}(x) = \frac{1}{2n} \sum_{i=0}^{2n-1} f_Y(x; h_n, p)$$

We can then integrate again to find the cumulative distribution function, which gives us the distribution of arrivals:

$$\begin{aligned} F_{total}(x) &= \frac{1}{2n} \sum_{i=0}^{2n-1} 1 - F_\omega\left(\frac{D}{y}\right) \\ &= 1 - \frac{1}{2n} \sum_{i=0}^{2n-1} \Phi\left(\frac{\frac{\sqrt{(h_c)^2+\left(\frac{p}{2}\right)^2}}{x} - \mu}{\sigma}\right) \end{aligned}$$

Once we have $F_{total}(x)$, we can then multiply it by the capacity C to find the total number of people who arrived at the stairs at time x. Furthermore, if we subtract from $F_{total}(x)$ the quantity $4x\beta$, the rate at which people leave the queue, then we have the length of the queue at time x. Finally, dividing this entire expression by 4β will give us the amount of time spent waiting when the individual at time x joins the queue.

$$F_{waiting}(x) = \frac{F_{total}(x) - 4x\beta}{4\beta}$$

The expected value of $F_{waiting}$ will be the average time spent waiting for someone who enters the queue, which is the quantity W in the numerical model.

The quantity $\overline{T}$ is computed by simply taking the expected value of $\frac{D}{\omega}$ which simplifies as follows:

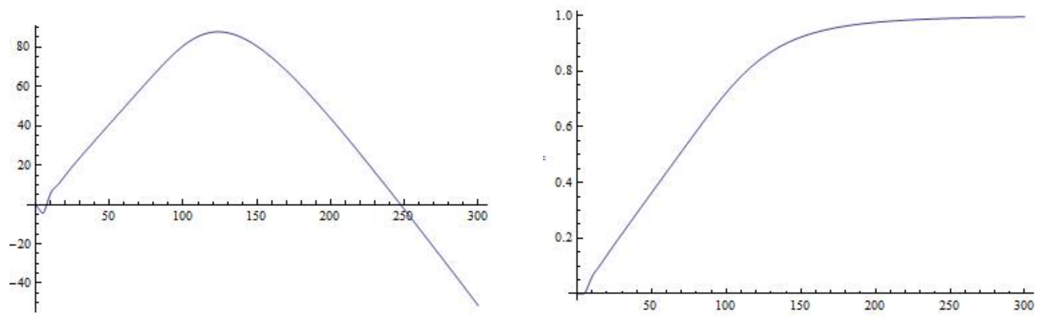

Figure 2: $F_{waiting}$ and F_{total} Graphs

$$\begin{aligned}\overline{T} &= E\left[\frac{D}{\omega}\right] \\ &= E\left[\frac{D}{\mu}\right] \\ &= \frac{1}{2n}\sum_{i=1}^{2n}\frac{\sqrt{(h_c)^2+\left(\frac{p}{2}\right)^2}}{\mu}\end{aligned}$$

Finally, S is computed in the same way as the numerical model which resulted in the expression:

$$S = \frac{5q}{8}$$

4.3.1 Example Model

Running this simulation through Mathematica using the example parameters used in section 4.2.1, we arrive at the following graphs for F_{total} and $F_{waiting}$

5 Results

5.1 Evaluation of Configurations

The computer program ran 100 runs for the each configuration and gave an average based on the staircase orientation and car carrying capacity. The data was based on the values shown in the example earlier in the section. The numbers in parentheses are used to represent where on the platform the stairways are placed. For example, 2 Stairways (1/4, 3/4) means that the stairways were placed $\frac{1}{4}$ of the way from one end, and $\frac{3}{4}$ of the way from the same end. All of the staircases are centered along the middle of the platform. The third column represents the required Q needed so that the last person who reaches the stairs would not have to wait. This was done by simply taking the total capacity and dividing by $2\overline{T}$. We called this Q_{req}.

$$Q_{\text{req}} = \frac{C}{2\overline{T}}$$

Type of Stairway	Average Time to Staircase (sec)	Q_{req} (people/sec)	Wait Time (sec)
1 Stairway (End)	77.304	6.467	52
1 Stairway (Middle)	39.530	12.648	90
2 Stairways (Ends)	39.426	12.682	45
2 Stairways (1/4, 3/4)	20.695	24.167	54.5
3 Stairways (Middle and Ends)	20.807	24.030	36.3
3 Stairways (1/6, 1/2, 5/6)	14.622	34.195	38.3

Table 2: Average waiting time and time to staircase values, including Q_{req}.

To evaluate the separate configurations in the table above, all factors were put in consideration to find the best configuration. As stated earlier in the section, the total time to leave the station is the sum of the walking time, the time waiting in the crowd, and the time on the stairs. However, the time on the stairs is practically negligible because it is the same amount for every passenger; all passengers are assumed to move up the stairs at the same rate. From our general equation, we would sum average wait time and the average time to the staircase and some constant for S, which is based on the number of stairs, q. We decided to use a "crowdedness" metric by using Q_{req} and dividing by 4β. We divided by 4β to account for the fact that 4 people/sec is the stairway up speed, so the total number of people per second is 4 multiplied by the number of stairways. Let M be this metric. From the values of the time to reach street level, we can easily see that the number of stairways generally decreases the amount of total time needed to exit the station. Based on how M was calculated, there would be a significant change if k varied. Because k was used to calculate the "crowdedness" metric, it shows that the crowdedness will decrease for every increasing value of k. Because it is in the denominator, the ratio will decrease with larger values and increase with smaller values.

From our data, we believe the configuration with stairways located 1/6, 1/2, and 5/6 along the platform would create the situation in which the average time to leave the station is the lowest. This is purely based on time, but we should also consider the excessive crowding that occurs at the bottom of the staircase. The configuration with two stairways at the end have the least crowding because some of people are already on the stairs while passengers farther away are just arriving at the staircase. With this in mind, we would recommend using the configuration with 2 stairways located at each end of the platform. Though the time is about 30 seconds higher, there would be a much smaller crowd, which implies there is a higher probability that the passenger would be spending more time walking to the stairs, instead of waiting. Walking to the stairs would

Type of Stairway	Time to Reach Street Level (sec)	M
1 Stairway (End)	$129.304 + \frac{5q}{8}$	1.617
1 Stairway (Middle)	$129.530 + \frac{5q}{8}$	3.162
2 Stairways (Ends)	$84.426 + \frac{5q}{8}$	1.585
2 Stairways (1/4, 3/4)	$75.195 + \frac{5q}{8}$	3.021
3 Stairways (Middle and Ends)	$57.107 + \frac{5q}{8}$	2.002
3 Stairways (1/6, 1/2, 5/6)	$53.922 + \frac{5q}{8}$	2.549

Table 3: Total time to reach street level and "Crowding" Metric

be a "better use" of a passenger's time. If it is necessary to have only one staircase, our model recommends positioning it at one end of the platform.

5.2 Analytical Solution

Type of Stairway	Average Time to Staircase (sec)	Wait Time (sec)	M
1 Stairway (End)	68.742	49.770	1.818
1 Stairway (Middle)	34.932	87.702	3.578
2 Stairways (Ends)	34.932	51.757	1.789
2 Stairways (1/4, 3/4)	18.308	41.279	3.413
3 Stairways (Middle and Ends)	18.308	56.037	2.276
3 Stairways (1/6, 1/2, 5/6)	12.719	55.697	3.276

Table 4: Data retrieved from the Analytical Model

This model has values of $\overline{T}$ which are generally always larger than the average which results from that of the numerical model. This is most likely due to the fact that the reciprocal of a normal distribution is skewed. In addition, W the waiting times for the configurations with 3 stairwells are much higher than that of the numerical model due to skew. In the analysis of the numerical models we determined that the distribution of arrivals to the staircases was fairly linear. However, in the analytical approach much more skew comes into play and so the heavier initial queues creates discrepancies. Thus the larger surges in the modeling of configurations with three stairwells creates longer average waiting times than the numerical approach and the trailing off in the modeling of the configurations involving one stairwell resulted in lower average waiting times.

Despite these discrepancies, the conclusions from this model are the same as the numerical model. As before, if we prioritize the speed at which people can exit the building, the configuration of stairways located at 1/6, 1/2, and 5/6 would allow for the minimum time spent walking out of the platform and station. However, if we are concerned with the crowding at the entrance of the stairs, we advocate the configuration with two stairwells at both ends of the platform. Finally, if the station has already been built or we are for some reason constrained to one staircase, then we recommend placement at the end since the total times are the same and crowding is minimized.

6 Strengths

- Our model accounts not only for minimizing the time needed for a passenger to get to street level, but also the time that passengers are left standing, which is more undesirable and unsafe than having the passengers walking for longer.
- By using two methods, we've increased the amount of data one can take into consideration when deciding upon a platform configuration. Presenting both models evaluates the accuracy of the other model, and gives two different approaches to the same problem.
- By using a normal distribution for the walking speed of passengers, we accounted for a wider range of commuters. This distribution would help in giving a more accurate representation of the people walking, allowing us to have a better represent the total time it takes for the passengers to reach the stairwell.

7 Weaknesses

- Our model doesn't account for pre-existing crowding in the platform that could impact the speed at which people can get to and climb the staircase. This could cause the time for people to leave the station to increase, and it is more difficult to model a pre-existing crowd. The model also assumes all passengers walk in a straight line to the stairs, but if there are other commuters, it will undoubtedly make passengers move in different directions.
- Though the walking speed is for a normal distribution, it may not be completely representative of everyone walking up the stairs. The speed for going up the stairs may not be accurate, as the amount of people per second would be different based on traffic, and the time of day.

8 Sensitivity Analysis

Independent Variable Change (%)	Dependent Variable Change (%)
+10% Q (speed going up stairs)	- 21.0% W (wait time)
-10% Q	+ 26.9% W
+10% q (number of stairs)	+ 10% S (time spent on stairs)
-10% q	- 10% S

Error in our Q value would have a significant effect on W, which directly affects our T_{total} total value. Because S is directly proportional to q, an error in the q value would have a proportional effect on the S value.

Change in n (Number of Cars)	Change in d (Length of each Car)	Change in C (Capacity of Train)	Effect upon $\overline{T}$ (Average arrival time)
+ 50%	- 33.334%	0%	+ 1.2% (negligible)
- 50%	+ 50%	0%	+ .26% (negligible)
+ 10%	0%	+ 10%	+ 9.2%
- 10%	0%	- 10%	-11.1%

Change in the n or d values without changing the overall capacity does not have a significant effect on the $\overline{T}$ value, but changing the overall capacity has a directly proportional effect on $\overline{T}$.

9 Future Applications and Extensions

Our model currently has general assumptions, and many factors are taken into account. However, there are some that we were not able to integrate into our models. For example, we would like to see how introducing a crowd to be on the platform initially would affect the time for a passenger to leave the station. Some of the people on the platform may want to exit, while others may attempt to get on the train at the same time passengers are getting off. It is also relatively unreasonable for a fully occupied train to have all of its passengers to leave the train at the same station. As mentioned before, some may transfer to other trains, stay on the same one, or loiter, so the crowd attempting to exit would not be as packed.

Another extension of this project we could pursue is to look at how different train setups would affect the timing. We would also investigate how the time it takes to reach street level varies based on time of day. Generally, the mornings and evenings would have the more traffic compared to the afternoon. This would cause the time to reach street level to be higher during the earlier and later parts of the day. Including time of day and taking into account varying traffic would easily change our distributions and calculated values.

10 Conclusion

Looking at the results from our metrics, we find that placing the staircase at the ends of the platform is best for minimizing both total time and time spent waiting for use for the stairs. When adding a second staircase the rule holds, and it's best to have those at the ends. Finally, when adding a third staircase, it becomes optimal to place them at $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{5}{6}$. We also explored the effects of changing the other variables and initial conditions in the model to see the effects, such as adding a second train or how many people the staircase(s) can accommodate. To see how these changes affect the total time that it takes to get to the street.

11 References

Sources used:

"R143 (New York City Subway Car)." Wikipedia. Wikimedia Foundation, 22 Oct. 2014. Web. 15 Nov. 2014.

Levine, Richard. "Seeking Bearable Level Of Subway Discomfort." New York Times. The New York Times Company, 10 Oct. 1987. Web. 15 Nov. 2014. http://www.nytimes.com/1987/10/10/nyregion/seeking-bearable-level-of-subway-discomfort.html .

Daamen, W., and S. Hoogendoorn. "Free Speed Distributions — Based on Empirical Data in Different Traffic Conditions." Springer Link. Springer Science+Business Media, 2007. Web. 15 Nov. 2014. http://link.springer.com/chapter/10.1007/978-3-540-47064-9_2 .

Shah, Jiten, Gaurang Joshi, and Purnima Parida. "Walking Speed of Pedestrian on Stairways at Intercity Railway Station in India." Proceedings Of The Eastern Asia Society For Transportation Studies 9 (2013): Easts Info. 2013. Web. 15 Nov. 2014. http://easts.info/on-line/proceedings/vol9/PDF/P269.pdf .

12 Appendix

A program was made in the programming language Java™ that calculates the time each person takes to reach the stairwell. Relevant functions are documented below. Namely, the function that returns a random set of random normal speeds for each person `getSpeeds()`, the function that returns distance each person in their respective car doors must travel to reach the stairway `findDisatnce()`, the function that calculates the time it takes to arrive at a stairway `calcArrivalTimes()`, and the function that runs these processes 100 times to develop an average T_i, `main()` and `runMain()`.

```
// returns average of 100 runs to get representative average for parameters
public static void main(String[] args) {
    double[] averages = new double[100];
    double sum = 0;
    for (int i = 0; i < averages.length; i++) {
        sum += runMain(50);
    }
    double superAvg = sum / averages.length;
    System.out.println("The average of " + averages.length + " runs is " + superAvg + " minutes");
}

static double runMain(int capacity) {

    ArrayList<Double> walkingSpeeds = getSpeeds(capacity);
    System.out.println(walkingSpeeds);
    int[] openDoors = {20, 40, 80, 100, 140, 160, 200, 220, 260, 280, 320, 340, 380, 400, 440,
         460, 500, 520, 560, 580};
    double[] stairways = {0.0};
    ArrayList<Double> distances = findDistance(openDoors, stairways);
    ArrayList<Double> arrivalTimes = calcArrivalTimes(distances, walkingSpeeds);
    System.out.println(arrivalTimes);
    double sum = 0;
    for (Double time : arrivalTimes) {
        sum += time.doubleValue();
    }
    double average = sum / arrivalTimes.size() / 60;
    System.out.println("Average = " + average);
    return average;
}

// returns list of individual people speeds that will
// be a random numbers within a normal distribution
// mean: 4.396; stdev: 1.213
static ArrayList<Double> getSpeeds(int capacityPerCar) {
    ArrayList<Double> speeds = new ArrayList<Double>();
    Random rand = new Random();
    for (int i = 0; i < capacityPerCar; i++) {
```

```
        speeds.add(rand.nextGaussian() * 1.213 + 4.396);
        // nextGaussian() returns random normal numbers
        // with a stdev of 1 and a mean of 0
    }
    return speeds;
}

static ArrayList<Double> findDistance(int[] doors, double[] stairways) {
// Returns an ArrayList of the closest distance each car door is from its respective stairway
    ArrayList<Double> distances = new ArrayList<Double>();
    for (int door : doors) {
        ArrayList<Double> allDistances = new ArrayList<Double>();
        for (double stairway : stairways) {
            allDistances.add(Double.valueOf(Math.sqrt(Math.pow(25, 2) + Math.pow(door - stairway,
                2))));
            // calculates distance of car door from each stairway
        }
        distances.add(Collections.min(allDistances));  // people will flock to closest stairway
    }
    return distances;
}

static ArrayList<Double> calcArrivalTimes(ArrayList<Double> distances, ArrayList<Double> speeds) {
    ArrayList<Double> arrivalTimes = new ArrayList<Double>();
    for (Double tdistance : distances) {
    double distance = tdistance.doubleValue();
        for (Double tspeed : speeds) {
            double speed = tspeed.doubleValue();
            // assuming it takes 3/4 of a second for next person to exit
            arrivalTimes.add(distance / speed + 3/4 * (arrivalTimes.size()));
        }
    }
    Collections.sort(arrivalTimes); // sorts times from least to greatest
    return arrivalTimes;
}
```

The following functions were used develop the analytical solution. The code below was written in Wolfram *Mathematica*™.

```
f1[x_] := 1 - Erfc[1.9111*(1.34 - 9.76/x)]/2
f2[x_] := 1 - Erfc[1.9111*(1.34 - 14.38/x)]/2
f3[x_] := 1 - Erfc[1.9111*(1.34 - 25.55/x)]/2
f4[x_] := 1 - Erfc[1.9111*(1.34 - 31.43/x)]/2
f5[x_] := 1 - Erfc[1.9111*(1.34 - 43.34/x)]/2
f6[x_] := 1 - Erfc[1.9111*(1.34 - 49.37/x)]/2
f7[x_] := 1 - Erfc[1.9111*(1.34 - 61.45/x)]/2
f8[x_] := 1 - Erfc[1.9111*(1.34 - 67.50/x)]/2
```

```
f9[x_] := 1 - Erfc[1.9111*(1.34 - 79.63/x)]/2
f10[x_] := 1 - Erfc[1.9111*(1.34 - 85.71/x)]/2
f11[x_] := 1 - Erfc[1.9111*(1.34 - 97.86/x)]/2
f12[x_] := 1 - Erfc[1.9111*(1.34 - 103.94/x)]/2
f13[x_] := 1 - Erfc[1.9111*(1.34 - 116.10/x)]/2
f14[x_] := 1 - Erfc[1.9111*(1.34 - 122.19/x)]/2
f15[x_] := 1 - Erfc[1.9111*(1.34 - 134.46/x)]/2
f16[x_] := 1 - Erfc[1.9111*(1.34 - 140.45/x)]/2
f17[x_] := 1 - Erfc[1.9111*(1.34 - 152.63/x)]/2
f18[x_] := 1 - Erfc[1.9111*(1.34 - 158.72/x)]/2
f19[x_] := 1 - Erfc[1.9111*(1.34 - 170.90/x)]/2
f20[x_] := 1 - Erfc[1.9111*(1.34 - 176.99/x)]/2
g[x_] := (f1[x] + f2[x] + f3[x] + f4[x] + f5[x] + f6[x] + f7[x] +
    f8[x] + f9[x] + f10[x] + f11[x] + f12[x] + f13[x] + f14[x] +
    f15[x] + f16[x] + f17[x] + f18[x] + f19[x] + f20[x])/20
Plot[(g[x]*1000 - 4*x)/4,  {x, 0, 300}]
Plot[g[x], {x, 0, 300}]
```