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2014

17th Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

(Please attach a copy of this page to your Solution Paper.)

Team Control Number: 4880
Problem Chosen: B

Please type a summary of your results on this page. Please remember not to include the name of your school, advisor, or team members on this page.

SUMMARY:

In order to determine how severe the spread of the disease is, we began by classifying the disease based on its mode of transmission. This was done by researching the respective reproductive rates (R_0) of representative diseases for each major mode of transmission and matching the range of values found with the R_0 of the unknown disease based on the data given in the problem. Using this method, we determined the disease to be transmitted primarily through air, with a R_0 of 1.79, meaning that on average an infected person will pass it on to 1.79 other people. Using this value, we also determined that the disease is not contained since the number of infected individuals is still increasing since R_0 is greater than 1. If R_0 is less than 1, then the number of cases will slowly decrease and the disease will slowly disappear from the system because infected individuals are not generating enough new cases.

To model the spread of the disease through the population, we modified the SIR model to include a new factor *E*, which represents the number of individuals in the 'latency phase', which means that they are exposed and infected by the disease but are not showing symptoms yet. This model is known as the SEIR model. Since most diseases have an incubation period before symptoms are manifested, it is necessary to consider *E* when modeling the spread of the disease.

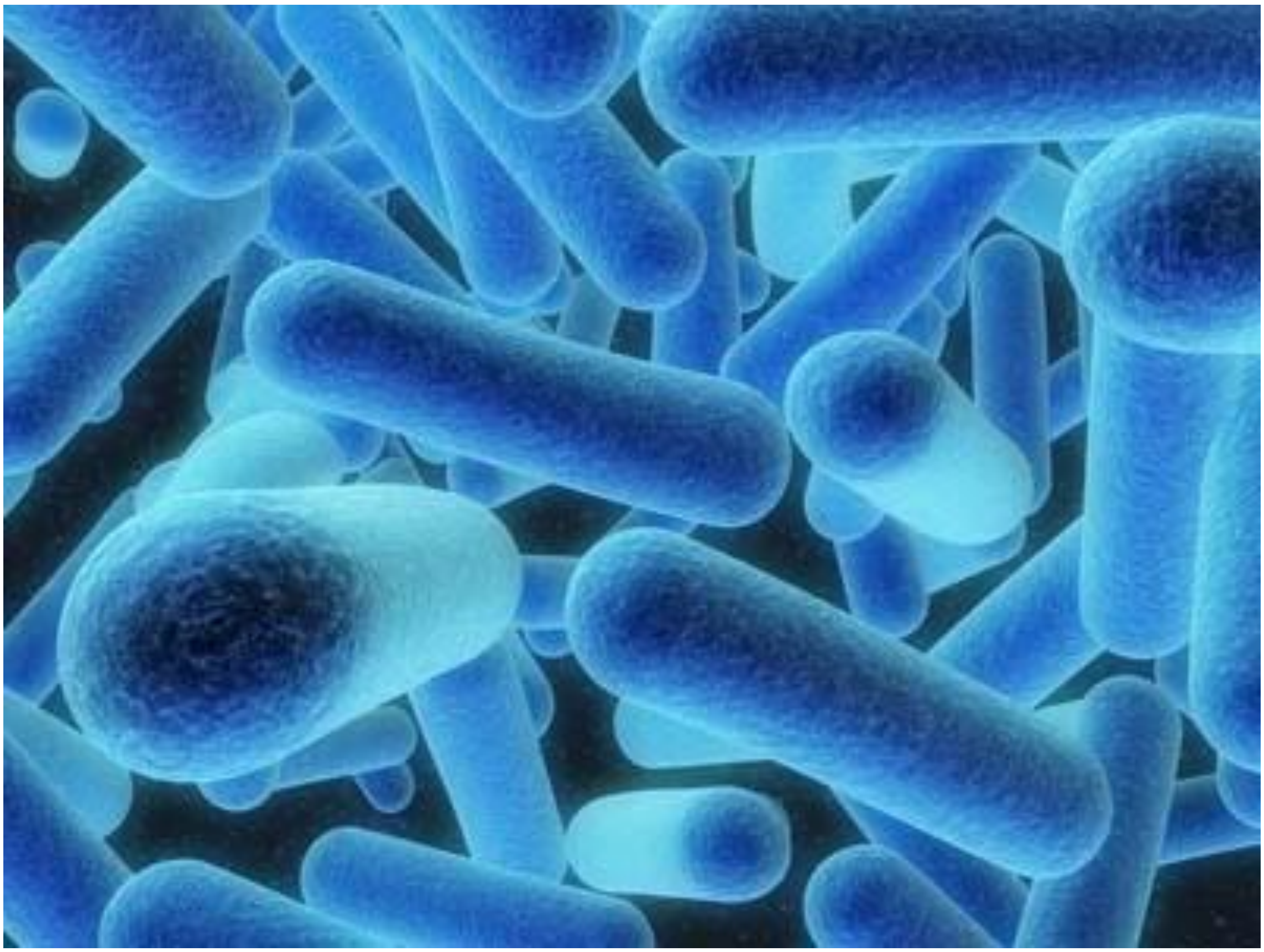
To determine the appropriate measures to be taken on the island, we first defined 6 different stages of a disease to match appropriate reactions and resource allocations. These stages are closely related to the variables in our SEIR model. I_1 represents the number of individuals that are not showing symptoms (S and E); I_6 represents the number of individuals that have died as a result of the disease (R); I_2, I_3, I_4, and I_5 represent the four stages of visible illness which have varying degrees of severity (I). These 6 variables were multiplied with respective constants that were determined by the amount of resources necessary for individuals in that stage, then added

together in a sum *W* which roughly represents, based on a scale, the amount of and types of resources an individual in certain stage of the disease will require.

However, these resources are not present in infinite amounts on the island, so we created another model to determine which countries could potentially provide such resources to the island. It is likely that Indonesia will provide a certain amount of aid to its own island, however it is also unlikely that it'll be able to provide all the resources needed to contain the disease. As a result, 12 other countries were selected due to their increased likelihood to provide aid for the island based on proximity to Indonesia and the general amount of supplies the country has in relation to Indonesia. The shorter the distance, the more convenient transportation of resources will be to the island, so the distance between the country and the island is defined as d. The general abundance of supplies is quantified by the ratio of GDP of the country to the GDP of Indonesia and is defined as R_{GDP}.

HiMCM 2014 Problem B:

The Next Plague?



Team #04880

November 6-7, 2014

Problem Summary and Analysis:

On a small island in Indonesia, a disease has begun to run its course by infecting 150 of the island's 300 inhabitants and killing 15. As a result, we have been asked to develop several mathematical models to determine and classify the pathogen, determine whether or not the disease is contained, determine at what point certain measures should be taken to prevent the spread of the disease and finally determine how to allocate resources to those on the island as a result of the change of the situation on the island.

Through the development of these models, additional situational information is provided to our team in order to help modify and improve the pre-existing models.

In this situation, people's lives are at stake, so the created mathematical models must accurately develop a solution that best prevents the disease from infecting and killing off the whole island. However, because this is an island, isolation is imperative in containing the disease and preventing its' spreading to other populations. As a result, the objective of these models is to quickly and efficiently allocate provided resources to those in different stages of the disease while taking into account where these resources are coming from as well.

Resources are not infinite supply though, so we identified nations that could provide help based on their GDP and number of physicians per 1000 people.

Model Design Justification:

In order to determine type and severity of spread, we had to determine method of transmission and how easily it could spread through a population, which we defined as the disease's basic reproductive rate . However, there was little information given in the problem, so we researched other diseases and used their information as a basis of comparison for the rate of transmission of our disease. We used a variation of the SIR model called SEIR, which included a pathogen's incubation period while assuming that recovered individuals could not be infected again, to calculate this disease's rate of of transmission. It was most similar to that of an airborne disease, so we assumed that this disease was also airborne.

Our final model could also be used to determine whether a disease was contained. We defined that a disease was contained when its basic reproductive rate fell below 1; this would mean that there will be fewer cases and the disease will eventually disappear from the population.

To create a model to determine the appropriate measures to be taken on the island, we first defined 6 stages of the disease, each with a varying degree of severity. The raw number of individuals in stage x, represented by I_x, was multiplied by a constant ($k, l, p, q, r,$ and v for stages 1, 2, 3, 4, 5, and 6 respectively) which was determined by how many resources an individual in that stage would need compared to someone in another stage. This is necessary because different stages of the disease require very different amounts of resources. For example, the constant p which corresponds to stage 3 is very high since a person in stage 3, which is more advanced and severe that stage 1, would require more attention and resources that someone in stage 1, so even the presence of a few individuals in stage 3 would greatly increase the amount of resources needed on the island. The sum of the variables multiplied by their respective constants is then divided by the number of individuals on the island minus the number of individuals who are not infected, which yields a value W(t) which is put in a scale to determine the overall severity of the disease on the island and what resources are necessary.

Variables

t = time

S(t) = S = Number of susceptible individuals who do not have the pathogen in their system

E(t) = E = Number of infected individuals who can not pass on the disease and do not show symptoms yet. In other words, the pathogen is in its latency period.

I(t) = I = Number of infected individuals who show symptoms of the disease, and can pass on the disease to others.

R(t) = R = Number of removed individuals who have been infected, but either died or recovered, so cannot pass on the pathogen.

N = Total population = S(t) + E(t) + I(t) + R(t)

D_L = Duration of latency (incubation), or the time between exposure to pathogens and show of symptoms

D_I = Duration of infection, or the time in days between show of symptoms and development of resistance/death.

R_0 = Basic reproductive rate of the disease. The number of secondary infections from one primary infection over the duration of the infection.

β = rate of transmission. The number of contacts made per day that result in transmission, or simply the number of new cases of infection from one original infection in one day.

m = mortality rate of the disease.

I_1, I_2, I_3, I_4, I_5, I_6 = Raw number of individuals in stages 1, 2, 3, 4, 5, and 6 of the disease, respectively.

W(t) = Rating value for quantity and type of resources necessary

k, l, p, q, r, v = constants multiplied to the size of each population group in determining W(t). Can vary depending on the location and disease.

A_P = Allocation of Physicians. Value used to compare the suitability of a country to send in physicians to that of other potential aid-providing countries and Indonesia.

A_S = Allocation of Supplies. Value used to compare the suitability of a country to provide desired supplies to that of other potential aid-providing countries and Indonesia.

R_P = Ratio of physicians per 1000 people of potential aid-providing country to that of Indonesia

R_{GDP} = Ratio of GDP of potential aid-providing country to that of Indonesia

Definitions

Type of spread: method of transmission

Severity of spread: how easily a given disease spread throughout a given population. Severity increases with the increase in β.

Containment: The disease is contained when there are no new infections. In other words, the number of cases is decreasing and R_0 is below 1.

Prevalent: Frequency is 0.5 or greater

Assumptions

1. The age structure of the population on the island is similar to that of Indonesia as a whole. *This is to ensure that the likelihood of death as a result of contracting the pathogen can be differentiated between age groups.*
2. Throughout the period, the crude birth rate (CBR) and crude death rate (CDR) are constant. *This ensures that there are no sudden differences in age structure or population size, and that all changes in population are due to the disease.*
3. The diseases chosen are representative of their method of transmission. For example, if cholera has the highest reproduction number, then water-born diseases are generally the most contagious. *It would be impossible to take all diseases of a particular method into consideration because there are thousands of them.*
4. The best way to classify method of transmission of an infectious disease is to compare of rate of transmission. *The transmission rate of diseases with same methods of transmission will spread at similar rates, while diseases of different methods of transmission will spread at clearly differentiable rates. In contrast, other factors such as Duration of Latency and Duration of Infection, however, can vary greatly within one method of transmission.*
5. There is only one pathogen on the island with the symptoms given. *This is to make sure that this disease will not be confused and therefore misdiagnosed as another disease.*
6. The virus will not mutate into a different strain. *If it did so, then it could have multiple methods of transmission and introduce too many factors into our model.*
7. The virus has a mortality rate of around 50%. *Since the symptoms of the disease are similar to those of ebola, it is reasonable to assume that the mortality rate will be similar as well. From the data provided, there is a 10% death rate, but it is reasonable to assume that there will be more deaths as the pathogen spreads throughout the population.*
8. The virus has an incubation period between 5-17 days. *Based on data collected, the average minimum incubation period is about 5 days, and the average maximum incubation period is about 17 days. Therefore, the time period is between the average minimum and average maximum.*

9. The 15 people who died as a result of the virus are the first to do so. *This is because it was not stated within the problem that there were people who died prior to this week, and it would be wrong to assume so.*

Requirement I: Disease Classification, Containment, and Resource Allocation Models

Methods of Transmission

Table 1. Representative Diseases of Methods of Transmission

Disease	Method of Transmission	R_0	Duration of Latency (D_L)	Duration of Infection (D_I)
Diphtheria	Airborne	6-7	2-5 days	4 days - 4 weeks
Measles	Airborne	12-18	7 - 21 days	8 days
AIDS/HIV	Bodily fluids (blood, urine, semen)	2-5	2 - 4 weeks	From point of infection, so could be 10+ years.
Ebola	Bodily fluids (blood, urine, semen)	1.5-2	2 - 21 days	Patients are infectious as long as they have the virus in their bodily fluids, which could be up until 7 weeks after recovery.
Cholera	Waterborne	1.0-2.75	2 hours to 5 days	7-14 days
Typhoid Fever	Waterborne	2.8	6-30 days	3-5 days (if treated) up to 1 month (untreated)
Dengue	Vector transmission	2.0-2.95	4-10 days	Virus lasts 1 month in body

SEIR Model:

$$\frac{dS}{dt} = \mu(N - S) - \frac{\beta SI}{N}$$

$$\frac{dE}{dt} = \beta\frac{SI}{N} - (\mu + \sigma)E$$

$$\frac{dI}{dt} = \sigma E - (\mu + \gamma)I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

μ for the given situation is roughly 1.64 x 10^{-5}. The crude death rate for Indonesia is 6 out of 1000 people per year, therefore the death rate per day would be:

$$\frac{6\ deaths}{1000\ people \times year} \times \frac{1\ year}{365\ days} = 1.64 \times 10^{-5} deaths/(people \times day)$$

R_0 is defined as the total number of secondary infections that originate from one primary infection over the entire time that an individual is infected. R_0 is calculated by the product of the total number of contacts made by infected individuals, the probability of transmission per contact, and the duration that a person remains infected. This means that dividing R_0 by the average duration of infection (D_I) will give us β, the rate of transmission.

R_0 can also be defined as $\frac{\sigma}{\sigma+\mu}\frac{\beta}{\gamma+\mu}$. However, in this case, $\mu = 1.64 \times 10^{-5}$, and so is negligible. Therefore, $\frac{\sigma}{\sigma+\mu}\frac{\beta}{\gamma+\mu}$ can be simplified to $\frac{\beta}{\gamma}$. $1/\gamma$ is the duration of infection (D_L), for individuals will move from the Infected to the Recovered at a relatively constant rate over the period of Infection. So, the equation can be rewritten as $R_0 = \beta D_L$, and again as $\beta = R_0/D_L$. This is consistent with the definition in the paragraph above. Therefore, it is logical and justifiable to consider:

$$\beta = R_0/D_L$$

The duration of latency (D_L) and rate of entrance from E to I (σ) is related by $D_L = \frac{1}{\sigma}$, because the Exposed will be entering the Infected at a relatively constant rate over the incubation period.

All of the equations of the SEIR model calculates for the instantaneous rate of change of the size of each population group. The rate is an infinitesimal change in the size of the group divided by a infinitesimal change in time. However, all of the calculations below uses modified equations:

$$\frac{\Delta S}{\Delta t} = \mu(N - S) - \frac{\beta SI}{N}$$

$$\frac{\Delta E}{\Delta t} = \frac{\beta SI}{N} - (\mu + \sigma)E$$

$$\frac{\Delta I}{\Delta t} = \sigma E - (\mu + \gamma)I$$

$$\frac{\Delta R}{\Delta t} = \gamma I - \mu R$$

This is an inevitable modification due to the lack of information given. The modifications will be further discussed in the sensitivity analysis.

$$\frac{\Delta R}{\Delta t} = \gamma I - \mu R$$

For this case, the current dR/dt is around 15/7, because 15 people died, and thus are no longer able to transmit the pathogen, over one week. The number of deaths due to the, disease can be calculated as $m \times R$. There were 15 deaths and since m was assumed to be 50%, R would be 30 and μ= 1.64 x 10^{-5}. I = 150, because there are currently around 150 individuals known to be showing symptoms of the disease.

After substitution: $\frac{15}{7} = 150\gamma - (1.64\times10^{-5})(30)$

With this equation, γ can be calculated.

γ= 0.0143

$$\frac{\Delta I}{\Delta t} = \sigma E - (\mu + \gamma)I$$

γ was calculated above as 0.0143. μ is unchanging, so μ= 1.64 x 10^{-5}. I =150. The time elapsed since the emergence of the Infected population was assumed to be 7 days, because that is when the first deaths occurred, proving the emergence of symptoms.

After substitution: $\frac{150}{7} = \sigma E - (1.64\times10^{-5} + 0.0143)150$

With this equation, σE can be calculated.

σE=23.576

$$\frac{\Delta E}{\Delta t} = \frac{\beta SI}{N} - (\mu + \sigma)E$$

The latency period of a disease, according to the research done on numerous different diseases, is typically not longer than 50 days (with the exception of AIDS). Therefore, the value of σ, which is equal to D_I^{-1}, would not be less than 0.02. As a result, μ is relatively negligible to σ. So, the equation above can be simplified to:

$$\frac{\Delta E}{\Delta t} = \frac{\beta SI}{N} - \sigma E$$

$\sigma E$was calculated above to be 23.576. I =150, and N = 300.

After substitution: $\frac{\Delta E}{\Delta t} = \frac{1}{2}\beta S - 23.576$

The $\sigma$for each of the disease researched can be calculated by D_L^{-1} by definition. Also, time interval of interest (Δt) would be the duration of latency (D_L) because the people that are currently part of the Exposed would have entered the group is the last D_L days. Therefore,

$\Delta t = \sigma^{-1}$. The σE for this case was determined to be 23.576. Exposed population (E) can be determined by $\frac{\sigma E}{\sigma}$, or $\frac{23.576}{\sigma}$. $\frac{\Delta E}{\Delta t}$ can then be expressed as $\frac{E}{\sigma^{-1}} = \sigma E = 23.576.$

Variables were defined so that $S(t) + E(t) + I(t) + R(t) = N$. This equation can be rewritten for this case to $S(t) + E(t) + 150 + 30 = 300$, and then to:

$$S(t) = 120 - E(t).$$

With $\frac{dE}{dt}$ and S values, the equation $\frac{dE}{dt} = \frac{1}{2}\beta S - 23.576$ can be used to write another expression and obtain another value for β.

Table 2. Diseases and their respective rates of transmission (β) from research

Disease	β **(R_0/D_I)**	σ**(D_L^{-1})**	**Average** σ
Diphtheria	0.214 - 1.75	0.2-0.5	0.35
Measles	1.5 - 2.25	0.0476-0.143	0.0953
AIDS	5.48×10^{-4}-1.37×10^{-3}	0.0357-0.0714	0.0536
Ebola	0.0306 - 0.0408	0.0476-0.5	0.274
Cholera	0.0714 - 0.393	0.2-12	6.1
Typhoid Fever	0.56 - 0.933	0.0333-0.167	0.100
Dengue Fever	0.0667 - 0.0983	0.1-0.25	0.175

Table 3. Diseases and their E, S and βvalues obtained from modified SEIR equation

Disease	E ($\frac{23.576}{\sigma}$)	S ($120 - E(t)$)	β= ($94.304/S$)
Diphtheria	67.36	52.64	1.79
Measles	247.387	<0 (so nobody is uninfected)	indeterminable
AIDS	439.851	<0 (so nobody is uninfected)	indeterminable
Ebola	84.044	35.956	2.62
Cholera	3.865	116.135	0.812
Typhoid Fever	235.76	<0 (so nobody is uninfected)	indeterminable
Dengue Fever	134.72	<0 (so nobody is uninfected)	indeterminable

Among all of the βvalue obtained from the modified SEIR equation (β= $94.304/S$), the βvalue was closest to the βvalue calculated from research ($\beta = R_0/D_I$) for Diphtheria. The βvalue from the modified SEIR equations for Diphtheria was 1.79, which is a 3.075% error from 1.75, the maximum βvalue for Diphtheria obtained from researched Base Reproduction Rate and Duration of Infection. The βvalue calculated also fits within the range of research-based βvalues for measles. Diphtheria and measles are both airborne diseases. Therefore, we can classify the disease affecting the island of interested as an airborne disease.

Containment

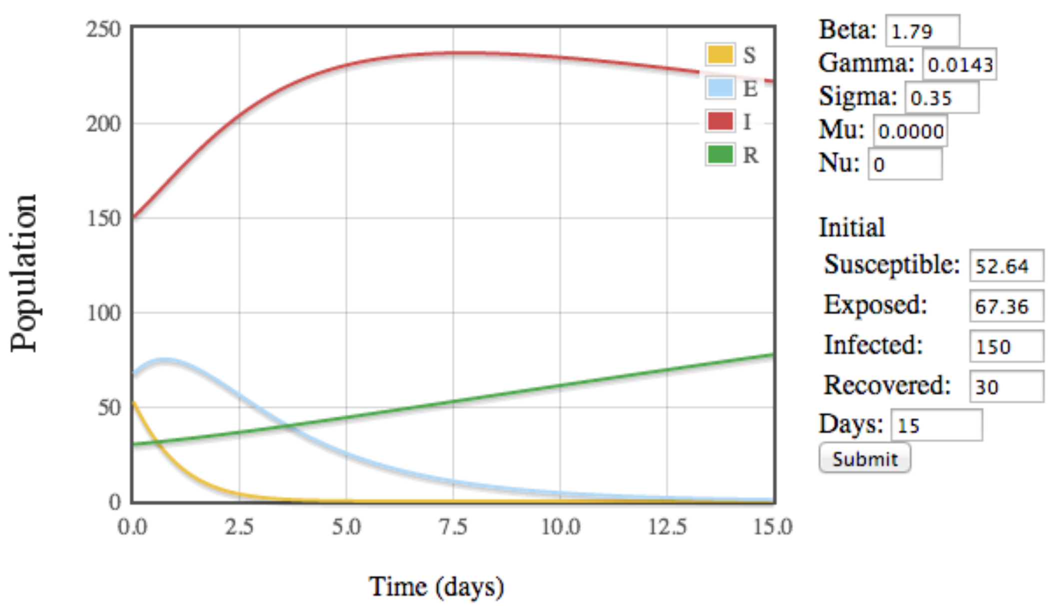
According to definition, a disease is contained if the R_0 values is less than 1, and not contained if the R_0 value is greater or equal to 1. Also, as derived previously,

$R_0 = \beta D_I$.

β value for the disease at hand was calculated to be 1.79. Using the researched D_L values for Diphtheria and Measles, the R_0 value ranged from 7.19 to 50.12. The entire range is greater than 1, and so the disease at the moment can be classified as uncontained.

When R_0 is greater less than 1, the average number of secondary conditions from one primary infection is less than one. This in turn means that the number of new infections is less than the existing infections, and that the number Infected individuals moving into the Removed exceed the number of Exposed individuals moving into Infected. Therefore, when R_0 is less than 1, the Infected population is decreasing.

Figure 1. Susceptible, Exposed, Infected, and Removed population over the next 15 days



The graph above predicts the population of each group over time. The curves were drawn based on the S, E, I, R, β,γ,σ,μ values determined from the analysis with the SEIR model above. t = 0 was assigned to be the current date. The curve of interest in the Infected curve, shown in red. As can be clearly seen, the Infected population is predicted to continue to increase at a decreasing rate, flatten out, then decrease afterwards. The Infected population starts to decrease at around 7.5 days. Therefore, it can be predicted that the disease will be contained after around 7.5 days from now.

Needs and Aids Assessment Model

Stages of disease

There are six stages of disease that require different distribution of resources: incubation, prodromal, illness, decline, convalescence and death. Incubation (I_1) is the period of disease in which a patient shows no visible symptoms, but has already been infiltrated by the pathogen. This stage requires distribution of personal hygiene products, distribution of medical masks and societal distance. Prodromal (I_2) is the onset of early symptoms of the disease. In this stage, it is best to immediately quarantine patients to prevent any further spreading of the disease. Furthermore, during this stage, it is important that medical personnel are at hand in order to correctly diagnose the symptoms as well as treat the patient. After doing so, it is also necessary that medicines are brought in in order to lessen the pain of the symptoms of disease. Illness (I_3) is a period of sickness that affects the mind or body and is the stage in which clinical symptoms appear. During the onset of illness, patients will need to remain in quarantine and will still require daily checkups by medical personnel as well as require the aforementioned medical supplies. Decline (I_4) is defined as the slow, progressive deterioration in health and vigor as a result of a specific disease. During this stage, patients need to continue to remain in quarantine so as to prevent the spreading of the disease. However, because they are in decline, they require less medical personnel to treat them as they are already unlikely to survive. Conversely, convalescence (I_5) is the period of disease at which the symptoms begin to fade until the time the patient recovers from the disease. Although the pathogen is not completely gone from the patient, they will still require medical attention and medical supplies in order to build up their health and strength. Furthermore, patients in this stage will still require to be kept in isolation, but whether or not they need to remain in quarantine will depend on each specific case. Finally, death (I_6) is the last possible stage. Here, a morgue is a required in order to prevent spreading of the disease from the dead body to others.

Implementation/Resources/Aid required for treatment of airborne disease in various stages:

As mentioned earlier, there are quite a few types of resources required to treat such a disease.

Firstly, quarantine is defined as a place of isolation in which people that have been exposed to infectious disease are placed. This is necessary for all the infected population (especially those in prodromal and illness stages) and might be needed for those in recovery as well.

Increase in societal distance is defined as a restricting movement and contact with the general population. This includes shutting down schools, restricting travels and imposing curfews on the general populace. By limiting the number of interactions with others, this decreases the possibility of a citizen contracting the disease.

Medical personnel in this situation refers to those involved in medicine or a specialist who has had prior experience with such diseases. Examples include doctors, nurses and trained specialists.

Medical supply can include both general and specific resources. Basic first aid kits should be brought in for the general population so that when certain symptoms appear, citizens will be able to use the kit to determine whether or not such symptoms are congruent with those of this disease. Furthermore, medicines specific to this disease that speed up process of recovery, are possible cures or lessen the severity of the symptoms would be required for each stage.

Personal hygiene products should be distributed to the population in order to further prevent the probability of the disease spreading to more of the population. Such products would include hand and body soap, shampoo and other products to keep the population as clean as possible. Other things such as medical masks and gloves should be distributed as a precaution as well.

Finally, the island as a whole will require the essential resources for survival. In order to contain this disease to this island and prevent it from spreading to the mainland, the island needs to isolate itself from other civilization. This would therefore require an increase in resources, as much of those basic needs were covered by trading with nearby islands. This would include food, water, clothing and other basic necessities.

A mathematical model below gives a rating value that can be converted to assess the quantity and type of resources needed at a given time.

$$W(t) = \frac{kI_1(t) + lI_2(t) + pI_3(t) + qI_4(t) + rI_5(t) + vI_6(t)}{N - S}$$

$I_1(t)$, $I_2(t)$, $I_3(t)$. $I_4(t)$, $I_5(t)$, $I_6(t)$ represent the raw number of individuals in stages 1, 2, 3, 4, and 5, 6 of the disease respectively. *k, l, p, q, r,* and *v* are constants, and are set across a very wide range, so that a slight increase or decrease in the population in each stage would not mistakenly indicate the quantity and type of aid necessary. The sum of the constant-population products are divided by the total population so that the size of the population does not affect the final result. These factors all combine into the sum *W(t)*, a rating value that can be converted to find the approximate quantity of each resource and type of aid that is necessary to be implemented on the island, based on the distribution of the population within the various stages at a certain time.

The specific values of the constants could vary for different diseases and locations, the following values were assigned to each constant for this case:

$k = 1$

$l = 10$

$p = 150$

$q = 100$

$r = 0.5$

$v = 0.1$

Different proportions of population were substituted into the equation to observe the results of W(t) depending on various scenarios:

Table 4. Possible W Values

Scenario Number	**I_1/N**	**I_2/N**	**I_3/N**	**I_4/N**	**I_5/N**	**I_6/N**	**W(t)**
1	1	0	0	0	0	0	1

2	0.9	0.1	0	0	0	0	1.9
3	0.9	0	0.1	0	0	0	15.9
4	0.9	0	0	0.1	0	0	10.9
5	0.9	0	0	0	0.1	0	0.95
6	0.9	0	0	0	0	0.1	0.91
7	0.8	0.2	0	0	0	0	2.8
8	0.8	0	0	0.2	0	0	20.8
9	0.8	0	0	0	0.2	0	0.9
10	0.8	0.1	0.1	0	0	0	16.8
11	0.8	0.1	0	0.1	0	0	11.8
12	0.8	0.1	0	0	0.1	0	1.85
13	0.8	0	0.1	0	0	0.1	15.81
14	0.8	0	0	0.1	0	0.1	10.81
15	0.8	0	0.2	0	0	0	30.8
16	0.8	0	0	0	0.1	0.1	0.86
17	0.7	0.1	0.1	0	0.1	0	16.75
18	0.7	0	0	0.2	0	0.1	20.71
19	0.7	0.1	0	0	0.1	0.1	1.76
20	0.7	0.2	0	0	0.1	0	2.75
21	0.7	0	0	0	0.3	0	0.85
22	0.7	0	0.3	0	0	0	45.7

23	0.7	0	0.1	0.1	0.1	0	25.75
24	0.7	0.3	0	0	0	0	3.7
25	0.7	0.1	0	0	0.2	0	1.8
26	0.7	0.2	0	0	0	0.1	2.71
27	0.6	0	0.1	0.1	0	0.2	25.62
28	0.6	0	0.4	0	0	0	60.6
29	0.6	0	0.2	0.1	0.1	0	40.65
30	0.6	0.1	0	0	0.2	0.1	1.71
31	0.6	0.2	0.2	0	0	0	32.6
32	0.6	0	0.1	0.1	0.1	0.1	25.66
33	0.6	0	0.3	0	0.1	0	45.65
34	0.6	0.2	0.1	0.1	0	0	27.6
35	0.6	0.2	0	0	0.2	0	2.7
36	0.6	0	0.1	0.1	0.2	0	25.7
37	0.5	0.2	0.1	0	0	0.2	17.52
39	0.5	0	0.5	0	0	0	75.5
40	0.5	0	0	0	0.5	0	0.75

41	0.5	0.2	0.1	0	0	0.1	17.51
42	0.5	0.2	0	0	0	0.3	2.53
43	0.5	0.2	0.1	0.1	0	0	27.5
44	0.5	0.3	0.1	0.1	0	0	28.5
45	0.5	0.4	0	0.1	0	0	14.5
46	0.5	0.3	0	0.2	0	0	23.5
47	0.5	0	0	0	0	0.5	0.55
48	0.4	0.2	0	0.3	0	0.1	32.41
49	0.4	0	0	0.5	0.1	0	50.45
50	0.4	0.1	0.1	0.3	0.1	0	46.45
51	0.4	0	0.4	0.1	0	0.1	70.41
52	0.4	0	0.4	0	0.2	0	40.5
53	0.4	0.3	0.3	0	0	0	48.4
54	0.4	0.5	0	0	0.1	0	5.45
55	0.4	0.5	0	0.1	0	0	15.4
56	0.4	0	0.2	0.1	0.1	0.2	40.47
57	0.4	0	0.2	0.1	0	0.3	40.43

58	0.3	0	0.1	0	0.2	0.4	15.44
59	0.3	0.2	0	0	0.4	0.1	2.51
60	0.3	0.3	0.3	0	0	0.1	48.13
61	0.3	0.1	0.2	0.3	0.1	0	61.35
62	0.3	0	0	0	0.3	0.4	0.49
63	0.3	0	0.3	0.4	0	0	85.3
64	0.3	0.1	0.1	0	0.4	0.1	15.51
65	0.3	0.1	0	0	0.1	0.5	1.4
66	0.3	0	0	0.3	0.2	0.2	30.42
67	0.3	0	0.4	0.3	0	0	90.3
68	0.2	0.1	0	0.4	0.3	0	41.35
69	0.2	0	0.5	0	0	0.3	75.23
70	0.2	0.4	0.4	0	0	0	64.2
71	0.2	0	0	0	0.7	0.1	0.56
72	0.2	0.3	0	0.5	0	0	53.2
73	0.2	0	0.8	0	0	0	120.2
74	0.2	0.1	0.1	0.1	0.5	0	26.45

75	0.2	0.7	0.1	0	0	0	22.2
76	0.2	0	0	0.6	0	0.2	60.22
77	0.2	0	0	0	0.4	0.4	0.44
78	0.1	0.5	0.4	0	0	0	65.1
79	0.1	0	0.9	0	0	0	135.1
80	0.1	0	0	0.3	0.2	0.4	30.24
81	0.1	0.2	0.2	0.1	0	0.4	42.14
82	0.1	0.1	0.1	0.5	0.2	0	16.25
83	0.1	0.4	0.4	0	0	0.1	64.11
84	0.1	0	0	0	0.6	0.4	1.34
85	0.1	0.2	0.3	0	0.4	0	47.3
86	0.1	0.7	0.2	0	0	0	37.1
87	0.1	0	0	0.2	0	0.7	20.17
88	0	0	0.4	0.5	0	0.1	110.01
89	0	0	0.1	0	0.8	0.1	15.41
90	0	0.6	0.2	0.1	0	0.1	46.01
91	0	0.3	0	0.4	0.1	0.2	43.07

92	0	0	1	0	0	0	150
93	0	0.1	0.1	0.2	0	0.6	36.06
94	0	0.9	0.1	0	0	0	24
95	0	0.2	0.2	0.2	0.2	0.2	52.12
96	0	0.1	0.6	0.2	0.1	0	111.05
97	0	0	0	0.1	0.8	0.1	10.41
98	0	0.5	0.2	0.1	0.1	0.1	45.06
99	0	0	0	0.3	0.6	0.1	30.31
100	0	0	0.5	0.5	0	0/	125

The table above calculates the W(t) values for various random scenarios. One hundred scenarios were generated so that the W(t) values can be seen, and the approximate range of W for the prevalence of different stage in the population could be observed. The range of W values shown in Table 5 was determined through observation of the values in the table above. As will be discussed later in the Sensitivity Analysis section, the table also serves to check the effectiveness of the constants in calculating a useful W(t) value.

Table 5. Conversion Table of W(t) into Resources Most Necessary

Range of W values	Resources Most Necessary
1-25	Increase in societal distance Distribution of personal hygiene products Some medical personnel Distribution of first aid kits

10-50	Quarantine Basic Necessities* More medical personnel
75-150	A lot of medical personnel Quarantine and isolated treatment wards Basic Necessities*
50-100	Medical personnel Medicine (pain control) Basic Necessities* Quarantine
0.5-5	Basic Necessities*
0.1-0.5	Quarantine, Morgue

*In this case, basic necessities refers to food, water, and safe shelter.

The table above allows for the conversion of the W(t) values into a list of the resources most necessary and their relative quantity. As will be discussed in the sensitivity analysis, this conversion of W(t) into a set of resources necessary may allow for more effective import of resources than just intuitively making decisions.

For this case of outbreak on the island, the current size of each population was determined by the modified SEIR equation to be:

$S = 52.86$

$E = 67.36$

$I = 150$

$R = 30$.

This can be translated to the stages of disease as such:

$I_1 = E = 67.36$

$I_5 = 15$, because there are 30 Removed, and mortality rate is 50%

$I_6 = 15$

I_2, I_3, and I_4 cannot be determined, but we will assume them to be roughly equal to each other, and so $I_2 = I_3 = I_4 = 50$

So, the W(t) value for this outbreak can be calculated to be:

$$W(t) = \frac{67.36 + 10(50) + 150(50 + 100(50) + 0.5(15) + 0.1(15)}{300 - 52.86} = 52.864$$

W(t) falls between the range of 50-100. Therefore, it can be determined from the conversion table that medical personnel, medicine, basic necessities, and quarantine are most necessary.

Resource Allocation Model

Because the disease has spread to about half the population of the island, isolation is required in order to prevent the disease from spreading to other populations. It is also unlikely that a small island with only 300 inhabitants is self-sufficient in all basic necessities and medical resources. As a result, resources will need to be brought in from other sources in order to allow the unaffected population of the island to have a relatively normal lifestyle even as the disease runs its course, and the affected population to be taken care of. Resources include basic necessities such as food, water, general hygiene equipment, as well as medical supplies and medical personnel.

A portion of the total resources supplied to the island would most likely be from Indonesia, as the island is part of the country itself. However, other countries may be eager to provide aid — and it may be simply be more convenient and beneficial to obtain resources from other countries. This model is to determine which foreign country the island should receive aid from to maximize benefits received, and the proportion of the aid that should be received from the foreign country.

Although it is possible that other nations will provide aid as well, the potential aid-providers were narrowed down to twelve most likely nations. This was based on the proximity of the country to Indonesia, the frequency of trade between the country and Indonesia, and the frequency of humanitarian aid launched.

Two separate models were developed for non-human supplies and medical personnel. Because these two resources come from different sectors of industry, a country may be suitable for providing basic necessities and hygiene supplies, but may not have many well-trained doctors to spare.

Two factors that affect the suitability of providing non-human supplies is the location of the country relative to the island, and the abundance of desired supplies in the country relative to Indonesia. The relative location was quantified by the shortest linear distance (d) between the country and the island. The shorter the distance, the quicker and more convenient transportation of supplies would be. Also, when the distance increases by a certain factor, the convenience and efficiency of transportation will most likely decrease by a greater factor. The relative abundance of supplies was quantified by the ratio of the GDP of the country to the GDP of Indonesia (R_{GDP}). GDP, not GDP per capita, was used because the nature of the situation only requires consideration of the quantity of supplies that can be produced and sent in, not the distribution of the supply across the country's population. A greater R_{GDP} value indicates that country is more economically developed compared to Indonesia. A greater R_{GDP}, therefore means that the country will most likely be able to provide a greater quality and quantity of desired supplies. Therefore, Allocation of Supplies value (A_S) can be determined by:

$$A_S = \frac{R_{GDP}}{d^2}$$

Two factors that affect the suitability of sending in medical personnel is the location of the country relative to the island, and the number of physicians in the country relative to Indonesia. The relative location was quantified by the shortest linear distance (d) between the country and the island. The closer the country to the island, the quicker and more convenient transportation would be. Additionally, if the distance increases by a given factor, the convenience and efficiency of transportation will most likely decrease by a greater factor. The relative number of physicians was quantified by the ratio of the physicians per one thousand people of the country to that of Indonesia (R_P). A greater R_P value indicates the country most likely has more physicians that they could potentially send into the island. Therefore, Allocation of Physicians value (A_P) can be determined by:

$$A_P = \frac{R_P}{d^2}$$

Table 6. GDP, A_S, Physicians per 1000 people, and A_P values for potential aid-providing countries

Country	**GDP (billions of USD)**	**R_{GDP}**	**Physicians per 1000 people**	**R_P**
Indonesia	2389.0	1	0.3	1.
USA	16,768.1	7.019	2.4	8.
European Union	17,578.4	7.358	N/A	N/A
Australia	1052.6	0.441	3.9	13.
Brunei	30.0	0.0126	1.4	4.667
Cambodia	46.1	0.0193	0.2	0.667
Laos	31.6	0.0132	0.3	1.
Malaysia	693.6	0.290	1.2	4.
Myanmar	221.5	0.0927	0.5	1.667
Philippines	643.1	0.269	N/A	N/A
Singapore	425.3	0.178	1.9	6.333
Thailand	964.5	0.404	0.4	1.333
Vietnam	475.0	0.199	1.2	4.

The table above lists the R_{GDP} and R_P values of all of the potential aid-providers. The ratios were determined to be representative indicators of ability to provide desired supplies or physicians to the island. The location of the island is unknown, so the value of d cannot be calculated. However, once the location of the island is known, the R_{GDP} and R_P values in table to obtains the A_S and A_P values.

Requirement II: Initial Recommendation for CDC

The disease on the island is airborne and has a $\boldsymbol{\beta}$ value of 1.79, which means that each infected person will transmit the disease to approximately 2 people.

1. Currently, the disease is not contained so movement in and out of the island should be controlled. Boats and planes coming in should be monitored and all passengers should be screened for possible symptoms. This is to prevent the disease from contaminating those outside the island, which would make it harder to control.
2. Distribute personal hygiene products and first aid kits to all households to prevent the spread of disease as much as possible.
3. This is an airborne disease, so educate the general public about how to prevent infection. This can be done by distributing masks and encouraging people to wash their hands frequently. This will reduce spread because will people know to be more careful.

Requirement III: Questions for Multinational Research Team

1. When did symptoms of this disease first start showing?
    1.1. The time of initial exposure to the pathogen was assumed to be the D_L + 7 days ago. By asking this question, we can obtain an accurate initial exposure time. This in turn will allow a more accurate calculation of the transmission rate (β).
2. How far is this island from the closest neighboring island that can provide the resources required to contain this disease?
    2.1. By determining distance from the infected island to one from which we can get supplies from, we can determine how long it would take and how hard it would be to get resources and supplies to the island.
3. For those who have contracted the disease, in what stage of the disease are they already in?
    3.1. The W(t) value for this outbreak in Requirement I was calculated with E and S values estimated through the modified SEIR equations. By finding out the number of individuals in the Susceptible, Exposed, Infected, and Removed groups, we can accurately determine how quantity and type of resources that are required through the Needs Assessment Model.

Requirement IV: Changes to Model

1. Appears to spread through bodily fluids

If this disease appears to spread through bodily fluids, this means that the disease has a different method of transmission to the one we'd initially believed. As a result, this change would affect the values of both R_0 and β. Transmission rate of transmission through bodily fluids is most likely less than that of transmission through air. Therefore, this information will cause the R_0 and βvalues to be lower than initially assumed.

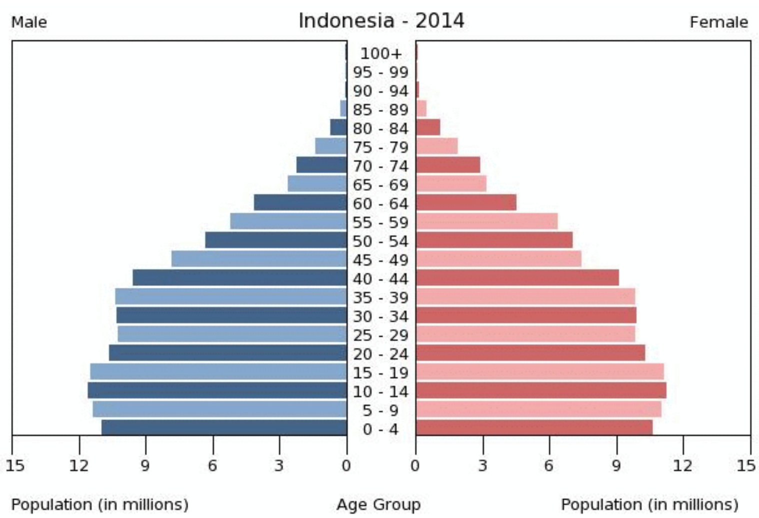
Furthermore, if the disease spreads through bodily fluids, different measures should be taken to contain the disease. Such measures could include a larger emphasis on masks, so that any bodily fluids that could exit through the mouth or nose are contained within the disposable mask. Stricter blood transfusion policies, or even the halting of blood transfusion while the disease are still prevalent in the population. Finally, an increase in the education and distribution of condoms. Although it is best for all members of the population to practice abstinence during this time period, the education of the use and distribution of condoms to the general populace will also help prevent the spread of the disease during coitus.

Finally, if the disease is spread through bodily fluids and isn't airborne, comparatively, there isn't as large a need for quarantine. This is because there is a slightly lower likelihood of contracting the disease for the average person on the island. As a result, this will decrease the values of p and q, and also narrow down the range of value W(t) that has a large quarantine requirement.

2. Elderly and children are more likely to die

The model could be adjusted to account for this new factor if the island population was divided into different age groups. Mortality rates for each age group would be very different, with those of the youngest and oldest age groups being the highest. Each rate could be used in our model to find W, and allocate resources. Indonesia is a developing country, so its age structure looks like a pyramid; there is a large young population base, and very few elderly people. As a result, it is important to minimize the mortality rate of the children to make sure that Indonesia does not experience a population crash.

Figure 2. Age Structure of Indonesia



3. Nearby island is showing signs of infection

Although the response model now applies to this island,the population of the nearby island cannot simply be added into the original model. Instead, a separate response model is needed for this situation because the other island has different population and health care characteristics. As a result, this separate response model, while based on the original equations, will provide different values because of the different populations, different number of people the disease has spread to, the proximity of said island to its own source for resources.

4. One of the researchers appears infected

The researchers would have known that the disease is prevalent and quite infectious, so would have been careful. The fact that still one of them got infected means that more careful measures need to be taken. Currently, it is known that the disease is spread through bodily fluids, so encourage masks, stricter policies on blood transfusion policies, and safe sex.

Furthermore, there is a higher transmission rate than expected—potentially larger $\beta$and R_0.

The response model now applies to the mainland as well. Number infected is 1, number exposed is undetermined. However, number exposed is undetermined but is potentially all those who were in close contact/proximity with the infected researcher.

So in order to prevent the situation from spiraling out of control, the researcher should immediately be put in quarantine and then the medical situation of the mainland should be assessed so that

Sensitivity Analysis:

1. Modification of SEIR Equations

The original SEIR equation calculates for the instantaneous rates of change in size of population. Due to the lack of information given, the SEIR equations were inevitably modified to calculate the average rate of change, not the instantaneous rate of change. The shortest time span given was one week, the period in which 15 deaths occurred. There was no given data for a narrower time span. Therefore, only the average rate, and not the instantaneous rate, of change in size of population groups could be determined.

This modification, without a doubt, resulted in expression giving estimations of values, and therefore the decrease in accuracy of many of the values determined using the SEIR equations. Still, the accuracy of the values obtained should still be considered quite high.
The practicality of application of the SEIR equations is questionable in real life. The number of new infections or deaths would most likely be measured in days, not in any smaller time intervals. Then, assuming that most data collection occurs daily in real life, the time interval that was used in the modified equation is not too large.

2. Assessment of Effectiveness of the Constants

Table 4, as well as providing many values of W(t) to be compared and used for determining appropriate ranges for conversion into most necessary resources (Table 5), assesses effectiveness of the constants in calculating a useful W(t) value.

The desired effect of the constants k, l, p, q, r, and v was to drastically increase or decrease the prevalence of each stage in the population. Ideally the range of W(t) value would be very large, but remain within a narrow range when the most prevalent (I_n/N 0.5) stage does not change. W(t) when different stages are prevalent should be differentiable so that ranges of W(t) can be attributed to resources most necessary.

The constants were assigned with consideration of the difference in magnitude, but were not any calculated value. The scenarios were, in a way, a test to see the effectiveness of the constants in distributing the groups of W(t) values for each prevalent stage across a wide range.

While the values of W(t) in Table 4 show some overlap and undefined boundaries between different most prevalent stage, it distributes the values across a wide range, and allows for the an effective range of W(t) to be assigned to each set of necessary resources.

3. Effectiveness of W(t) in Needs Assessment

The needs assessment can be done by simply observing how prevalent the infection is in the area, without calculating W(t). However, this model sheds light not only on the prevalence of the infection, but other factors, and thereby allows for a fuller and more effective needs assessment. The model divides the Infected population into various stages of disease. It recognizes that different stages of infection, within the Infected population, requires different quantity and types of resources. This allows for determination of resources most necessary for the population, and the subsequent import of only the most necessary resources.

The needs assessment also accounts for the needs of the stages of disease that require a lot of resources even if they are less prevalent. The large constants multiplied to the stage 3 and stage 4 ensures that even if only a small fraction of the population is in stage 3 or 4, the W(t) value will be large enough that the conversion will indicate the necessity of quarantine medical personnel.

Strengths and Weaknesses

Strengths:

1. The SEIR model was modified to fit our purpose and case. This ensured that the final model could be applied to the case, and that the data available could be substituted to find the desired values of variables.
2. E and S were not assumed; instead the they were estimated from other parts of the SEIR model and the given data. With less assumptions, we were able to find a model that more accurately fit the given situation.
3. We considered almost all possibilities when determining the possible W values. We realized that the population would not always have an equal amount of individuals in each stage of a disease, so we calculated all scenarios, numbered them, and then used a random number generator to select 100 distinct scenarios.
4. Multiple variables were considered when determining the allocation of physician and supplies value, and the variables were quantified using logical measures.

Weaknesses:

1. $\frac{dX}{dt}$ in our model was modified into $\frac{\Delta X}{\Delta t}$. Because the time frame given in the data was not specific enough to calculate the instantaneous rate of change of X (Either S, E, I, R), it was modified into $\frac{\Delta X}{\Delta t}$, which represents the change of the variable over time. Thus, the accuracy of the equation is compromised since $\frac{\Delta X}{\Delta t}$ only gives a rough estimate of how the variable is changing, rather than a specific rate of change which is what $\frac{dX}{dt}$.
2. When solving for β, the equation was simplified by eliminating the variable μ due to its negligibly small value. This inevitably creates a margin of error for the calculated value of β.

3. In the needs assessment rating model, the variable I_1 cannot be accurately gauged since individuals in the incubation stage have no visible difference with individuals who are not infected with the disease.
4. Mortality rate and time since exposure to infection were assumed. There was not enough information given in the problem, so we used the mortality rates and incubation periods to determine reasonable ones for us.

A Letter to a Local News Outlet

Dear Editors of South China Morning Post,

We are a group of researchers working for the Hong Kong Division of the International Health Organization. Last week, we were asked to travel to an Indonesian island to investigate an outbreak of an unknown disease that could be the start of an epidemic.

When we arrived, there was very little information on the disease itself, so we started by determining a rate of transmission: a measure of how easily a disease spreads throughout a population. By using a modified version of the Susceptible, Exposed, Infected and Removed (SEIR) model of epidemiology, which takes a disease's incubation period into consideration, and the basic reproductive rate, which is the average number of infected individuals a single case can drive, we were able to find a rate of transmission consistent with that of airborne diseases. Because an airborne disease can spread very quickly throughout a population, it is consistent with the fact that the disease had already infected about half of the existing total population of 300. This model also used the basic reproductive rate to determine whether a disease was contained; if the rate fell below 1, then the disease would slowly disappear from the population.

Another model was then created in order to determine which resources such as doctors, containment facilities, and medicine, would be needed depending on patient's condition. We determined that patients who were in the third and fourth stages of the diseases would need the most resources, simply because these people had the highest risk of death. In order to account for this, we came up with an equation that gave us a value representing the amount of resources each type of patient would need. The value would fall into one of six categories, which would decide the amount and type of resources the island would need in order to contain the disease. However, the island does not have an unlimited supply of such resources. As a result, we created another model that would allow us to identify countries that could potentially provide aid by using the Gross Domestic Product (GDP) of each of the identified countries, as well as their physician to 1000 citizen ratio. Although Indonesia is most likely able to and willing to provide a certain amount of resources to the island, it is unlikely that Indonesia will be able to cover all costs. As a result, it might be more beneficial for the island to receive aid from another country. There are 12 potential aid providers, and the suitability of each country to provide aid was determined by the GDP and the aforementioned ratio.

After further research was done by our multinational research team, it was discovered that the disease spread through exchange of bodily fluids, rather than through the air. In order to use this information, we altered our mathematical model so we could account for the different values. Our rate of transmission decreased, and as did the basic reproductive rate of the disease.

Therefore, there is no need for the general public to panic. Diseases spread through bodily fluids are much less contagious than those spread through the air, so this one poses a much lower risk to public safety. Although the disease has not been contained yet, it is important to remember that this is a new pathogen. It will take time to bring it under control.
Thank you for taking the time to read our report.

Sincerely,

Team 4880

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