The patient must be reached before medical attention can be given, making minimizing the medical response time to reach a zone of population a societal concern. The purpose of this problem is to analyze the average time it takes an ambulance to travel from each zone to any zone in the county in order to maximize the population reached in the smallest amount of time. Using these results, we will form a plan for the Emergency Service Coordinator to direct ambulances.

Our solutions depend on the number of available ambulances and the number of emergencies the county has to deal with at a specific time. When three ambulances are available, 11 out of 20 combinations of ambulance placement will result in perfect coverage of the entire county within eight minutes. To determine which of these combinations resulted in optimal coverage we defined a new unit, people-minutes, which refers to the total amount of time required to reach every individual given a single arrangement of ambulances. Using this unit, we found the average time required to reach an individual by finding the total minimum people minutes for a given ambulance arrangement and dividing this by the total population of the county. In essence, it weighted the time needed to reach different zones by the relative proportion of the population in that zone. We wanted to make sure that not only were our ambulances quickly reaching citizens, they were also maximizing the number of citizens they could reach. We determined that zones 1,2 , and 5 were the optimal locations for ambulance placement, as this combination had the lowest average time, only 2.37 minutes. In the case where only two ambulances were available, perfect coverage could still be achieved, with ambulances in zones 2 and 5 . With a single ambulance, it was impossible to cover every zone in eight minutes. We did determine that zone 2 would be the optimal zone for that single ambulance, because it would cover the most people within the time constraint. For each of these cases, we also considered situations where conditions may not be ideal and looked at the difference in coverage when the average travel time increased by ten percent. Our models were all relatively robust as a large proportion of the population could still be reached in eight minutes.

We also wanted to consider other realistic emergencies that could occur. We looked at the best plan of action for the ambulances when multiple emergencies occurred at once. We saw that people in zones 1 and 2 could be reached quickly while getting to zones 5 and 6 was less efficient. Overall, $87.25 \%$ of these situations can be successfully covered. If there were slowdowns on roadways resultant in an increase in time required to travel from zone to zone, two ambulances would not offer coverage within eight minutes to all parts of the county. In reference to the ability of three ambulances to respond to a disaster affecting multiple areas, we determined three to be insufficient in large-scale chaos, though it would be possible to ameliorate the situation by relying on other resources. At the end, we advised the Emergency Service Coordinator on what to do, especially focusing on the county's plans in case of a large-scale catastrophe. Our model hopes to improve the average county citizen's life by increasing the number of people that can be reached by the ambulances and decreasing the travel time it takes to reach them.

## Problem A: Emergency Medical Response Team 4421

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## 1 Introduction and Restatement of the Problem

For some patients, particularly cardiac arrest and shock victims, every minute is vital, and just one or two could make the difference between life and death. Therefore, the Emergency Medical Services have begun to place emphasis on driving the average medical response times to every area under eight minutes; it is a notably high standard to live up to but quite possible to achieve when taking into account the rapidity of the situation.

For this problem, we attempted to determine whether it was possible to service every zone in a county within eight minutes with different ambulance arrangements. We addressed situations in which one, two, and three ambulances were available. If it was possible, we sought to ascertain which combination of ambulance placements would result in the lowest average time to reach an individual. We then considered the ability of three ambulances to respond to widespread disaster in the county. The most important factors in determining ambulance placement were the times that the ambulance would reach different zones in and the amount of people the ambulance could reach in under eight minutes. We finished our paper by writing a letter to the emergency coordinator advising him or her on emergency precautions and design ideas to aid ambulances in future disaster response.

## 2 Definitions, Variables, and General Assumptions

### 2.1 Definitions

- We were given the Emergency Medical Response data for a county with six zones. We assume that this data will form a basis for our projections of continued population growth in this area.
- The average travel time is calculated as the time taken to get from the center of one zone to the edge of another. The emergency is dealt with once the ambulance reaches the zone.
- We will define coverage or success as the ambulance being able to reach every zone within the time limit of eight minutes.
- We define the probability of an emergency occurring in a zone as the percentage of the population in that zone.


### 2.2 Variables

- $\bar{t}$ is the expected average response time of the ambulance in minutes
- $t_{n}$ is the shortest ambulance response time to the respective zone in minutes
- $\quad p_{n}$ is the population of the respective zone
- $\quad P$ is the total population of 270,000 in the county


### 2.3 Assumptions

Assumption: Zones are internally uniform.
Justification: We were given no information on population density or other factors affecting the zones. There is no accurate way to predict how ambulances will be affected driving from or to the zone.

Assumption: Each person is independent and equally likely to face an emergency. Justification: This assumption provides the basis for our definition of an emergency occurring in a specific zone.

Assumption: Each ambulance can respond to any zone in the city.
Justification: In order to minimize the time taken to reach each zone, each ambulance must be available for travel to every zone that it is able to reach within the time limit. It is reasonable to conclude that whichever ambulance is available and minimizes the response time will be used, without regard to potential future emergencies.

Assumption: A single emergency will occur at once, and the ambulance sent will have returned by the next call.

Justification: Often, a single ambulance must be responsible for multiple zones in order to ensure coverage within eight minutes. Rutherford County in Tennessee has a population of 274,454 people, just a few thousand people above from our theoretical county. They receive approximately sixty-two calls per day, averaging out to about one call every twenty-three minutes when not including factors such as time of day (2). We
assume that this county is representative of our theoretical one and twenty-three minutes is ample time for an ambulance to reach the emergency site and return before the next emergency.

Assumption: Each ambulance will be placed in a different zone.
Justification: The previous assumption eliminates the demand for multiple ambulances in one zone because if an ambulance is stationed in a zone, it will always be available to address the next emergency for all of the zones it is assigned to.

## 3 Creating a Model

### 3.1 Three Ambulances

### 3.1.1 Determining Optimal Location

In order to create a realistic model for the three ambulances, we looked at all the possibilities for three ambulances to be placed in the county. It was important that between the three ambulances, each zone could be reached within eight minutes under semi-perfect conditions. Below, in Figure 1, we see that there are possibilities for each zone to be reached within the time limit.

Figure 1: Average Travel Time in Minutes for an Ambulance to Reach a Zone

|  | Average Travel Time (minutes) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zone | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| $\mathbf{1}$ | 1 | 8 | 12 | 14 | 10 | 16 |
| $\mathbf{2}$ | 8 | 1 | 6 | 18 | 16 | 16 |
| $\mathbf{3}$ | 12 | 18 | 1.5 | 12 | 6 | 4 |
| $\mathbf{4}$ | 16 | 14 | 4 | 1 | 16 | 12 |
| $\mathbf{5}$ | 18 | 16 | 10 | 4 | 2 | 2 |
| $\mathbf{6}$ | 16 | 18 | 4 | 12 | 2 | 2 |

In total, there are ${ }_{6} \mathrm{C}_{3}$ possible ways to arrange three ambulances in six zones with only one ambulance in each zone. This leaves only twenty different ways to arrange the ambulances. We tabulated all twenty different ways to arrange the ambulances. We then checked to see which possibilities would cover all six zones in less than eight minutes.

Figure 2: All Possible Locations for Three Ambulances

|  | Ambulance Zone |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Possibility | 1 | 2 | 3 | 4 | 5 | 6 | Success | Uncovered Zones |
| 1 |  |  | , |  |  |  | N | 4 |
| 2 |  |  |  | , |  |  | N | 5,6 |
| 3 |  |  |  |  |  |  | Y |  |
| 4 |  |  |  |  |  |  | N | 4 |
| 5 |  |  |  |  |  |  | Y |  |
| 6 |  |  |  |  |  |  | Y |  |
| 7 |  |  |  |  |  |  | N | 4 |
| 8 |  |  |  |  |  |  | Y |  |
| 9 |  |  |  |  |  |  | Y |  |
| 10 |  |  |  |  |  |  | Y |  |
| 11 |  |  |  | , |  |  | Y |  |
| 12 |  |  |  |  |  |  | Y |  |
| 13 |  |  |  |  |  |  | N | 4 |
| 14 |  |  |  |  | , |  | Y |  |
| 15 |  |  |  |  |  |  | Y |  |
| 16 |  |  |  |  |  |  | Y |  |
| 17 |  |  |  |  |  |  | N | 1,2 |
| 18 |  |  |  | $\bigcirc$ |  |  | N | 1,2 |
| 19 |  |  |  |  | $\times$ |  | N | 1,2 |
| 20 |  |  |  |  | < |  | N | 1,2 |

As shown above in Figure 2, there are eleven possible ways to place the ambulances in three different zones such that all the zones can be reached within eight minutes. Thus, it is possible to reach any member of the population within eight minutes of the emergency call.

However, we want to find the optimal three zones to place the ambulances in order to reach the population in the shortest amount of time. To do this, we created a new unit, termed people-minutes, referring to the total amount of time required to a reach each individual in the county. To define this variable we determined which ambulance would
have the shortest response time for each of the zones according to each of the twenty successful arrangements. After finding this, we multiplied the population by the response time. To calculate the average time needed to reach an individual, we divided by the total population of the county, 270,000 residents, thus determining the expected response time the ambulance would have for a random emergency call based on the population distribution in the county.

$$
\bar{t}=\sum_{n=1}^{6} \frac{t_{n} * p_{n}}{P}
$$

We calculated the expected average response time if ambulances were placed in zones 1,2 , and 5 . The italicized and underlined items in the table below show the shortest ambulance response time for each zone.

Figure 3: Average Ambulance Travel Time and Population Per Zone

|  | Average Ambulance Travel Time (minutes) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zone | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| $\mathbf{1}$ | $\underline{1}$ | 8 | 12 | 14 | 10 | 16 |
| $\mathbf{2}$ | 8 | $\underline{1}$ | $\underline{6}$ | 18 | 16 | 16 |
| $\mathbf{5}$ | 18 | 16 | 10 | $\underline{4}$ | $\underline{2}$ | $\underline{2}$ |
| Population | $\mathbf{5 0 , 0 0 0}$ | $\mathbf{8 0 , 0 0 0}$ | $\mathbf{3 0 , 0 0 0}$ | $\mathbf{5 5 , 0 0 0}$ | $\mathbf{3 5 , 0 0 0}$ | $\mathbf{2 0 , 0 0 0}$ |

If we input this data into the formula, we get the following equation
$\bar{t}=\frac{1(50,000)+1(80,000)+6(30,000)+4(55,000)+2(35,000)+2(20,000)}{270,000}$
This simplifies to give us an expected average response time of 2.37 minutes.

Following the same procedure for the other ten possible ambulance arrangements, we found the other expected average response times and were able to compare them below in Figure 4.

Figure 4: Expected Average Response Times for Arrangements of Three Ambulances

| Ambulance <br> Position | $\overline{\boldsymbol{t}}$ <br> (minutes) |
| :---: | :---: |
| $1,2,5$ | 2.37 |
| $1,3,4$ | 4.00 |
| $1,3,5$ | 3.94 |
| $1,4,5$ | 3.61 |
| $1,4,6$ | 3.61 |
| $1,5,6$ | 4.22 |
| $2,3,4$ | 3.22 |
| $2,3,5$ | 3.24 |
| $2,4,5$ | 2.83 |
| $2,4,6$ | 2.83 |
| $2,5,6$ | 3.44 |

The data shows that ambulances placed in zones 1,2 , and 5 will provide the shortest expected average response time for a member of the population, at only 2.37 minutes, which is well within the eight-minute limit. Since this arrangement successfully reaches all zones and maintains the lowest average response time, it is the optimal placement of the ambulances.

### 3.1.2 Sensitivity Analysis

To determine the robustness of our solutions, we tested what effect a change in average travel times would have on the success of the ambulances reaching an emergency in time. Because the data given to us represented average travel times under perfect conditions, it is reasonable to assume it could take slightly longer for the ambulance to reach the location during an actual emergency.

Figure 5: Increased Shortest Travel Times

|  | Average Travel Time (minutes) |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Zones | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |  |
| $\mathbf{1}$ | 1.1 | 8.8 | 13.2 | 15.4 | 11 | 17.6 |  |
| $\mathbf{2}$ | 8.8 | 1.1 | 6.6 | 19.8 | 17.6 | 17.6 |  |
| $\mathbf{3}$ | 13.2 | 19.8 | 1.65 | 13.2 | 6.6 | 4.4 |  |
| $\mathbf{4}$ | 17.6 | 15.4 | 4.4 | 1.1 | 17.6 | 13.2 |  |
| $\mathbf{5}$ | 19.8 | 17.6 | 11 | 4.4 | 2.2 | 2.2 |  |
| $\mathbf{6}$ | 17.6 | 19.8 | 4.4 | 13.2 | 2.2 | 2.2 |  |

Figure 5 shows how increasing the times by $10 \%$ would render many of our earlier possible solutions ineffective. An ambulance in zone 1 is the only one able to reach zone 1 in time. Similarly, an ambulance in zone 2 is the only one able to reach zone 2 in time and can also reach zone 3 within eight minutes. The only remaining ambulance must go in zone 5 to cover zones 4,5 , and 6 in eight minutes or fewer. Other than the optimal combination of zones 1,2 and 5 , no combinations can reach each zone in less than eight minutes.

Next, we used the adjusted $t_{n}$ values to calculate a new expected response time.
Figure 6: Increased Shortest Travel Times for Ambulances in Zones 1, 2, and 5

| Zone | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Response <br> Time <br> (minutes) | 1.1 | 1.1 | 6.6 | 4.4 | 2.2 | 2.2 |

The ten percent increase did not change the travel times by too much. All of the times are still under the eight-minute limit, with the longest travel time being an ambulance travelling from zone 2 to 3 in 6.6 minutes.

Using the same procedure as we did earlier, we found the new expected average response time of the ambulances in our optimal arrangement. It increased slightly to 2.61
minutes. Changing each of the travel times by $10 \%$ caused a $10 \%$ increase in the time as expected, because $t_{n}$ is linearly related to $\overline{\boldsymbol{t}}$. This shows our optimal solution to still be successful even with a ten percent increase in the travel time.

Our optimal solution would no longer be valid once a zone cannot be reached within eight minutes. Currently, zone 3 takes an the longest to reach under semi-perfect conditions, at an average of six minutes. We want to see what percent change there would have to be in the travel times for an ambulance to no longer reach zone 3 within the eightminute limit.

$$
\frac{8 \text { minutes }-6 \text { minutes }}{6 \text { minutes }}=0 . \overline{3}
$$

Average travel times would have to increase by over approximately $33 \%$ for our optimal solution to be unsuccessful.

### 3.1.3 Strengths and Weaknesses

Our optimal location prediction works well because it accounts for the population dispersion between the zones and the shortest travel time for an ambulance from zone 1 , 2 , or 5 to reach the emergency. All of the zones can be reached quickly. Our prediction also held up well under a sensitivity analysis. Because the average response time for the entire county is linearly related to the average travel times under semi-perfect conditions, a small change in the average travel times will not have a large effect on our model.

However, our model is overly simplistic in some areas. Because we have no information about the zones' population dispersals or sizes, we do not have more realistic interpretation to provide. Our model also assumes that there will be only one emergency at a time, which may not always be true.

### 3.2 Two Ambulances

### 3.2.1 Determining Optimal Location

There are only ${ }_{6} \mathrm{C}_{2}$, or 15 , different ways to place two ambulances in different zones. Once again, we tabulated the different possible arrangements.

Figure 7: All Possible Locations for Two Ambulances

|  | Zones |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Possibility | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | Success | Uncovered <br> Zones |
| $\mathbf{1}$ |  |  |  |  |  |  | N | $4,5,6$ |
| $\mathbf{2}$ |  |  |  |  |  |  | N | 4 |
| $\mathbf{3}$ |  |  |  |  |  |  | N | 5,6 |
| $\mathbf{4}$ |  |  |  |  |  |  | N | 3 |
| $\mathbf{5}$ |  |  |  |  |  |  | N | 4 |
| $\mathbf{6}$ |  |  |  |  |  |  | N | 4 |
| $\mathbf{7}$ |  |  |  |  |  |  | N | 5,6 |
| $\mathbf{8}$ |  |  |  |  |  |  | Y |  |
| $\mathbf{9}$ |  |  |  |  |  |  | N | 4 |
| $\mathbf{1 0}$ |  |  |  |  |  |  | N | 1,2 |
| $\mathbf{1 1}$ |  |  |  |  |  |  | N | 1,2 |
| $\mathbf{1 2}$ |  |  |  |  |  |  | N | $1,2,4$ |
| $\mathbf{1 3}$ |  |  |  |  |  |  | N | 1,2 |
| $\mathbf{1 4}$ |  |  |  |  |  |  | N | 1,2 |
| $\mathbf{1 5}$ |  |  |  |  |  |  | N | 1,2 |

There is only one possible arrangement of the two ambulances that covers the entire county. The ambulances would have to be in zones 2 and 5 in order to reach every resident in eight minutes or less. Again, our optimal solution lies with the assumption that the ambulances will only deal with one emergency at a time.

We used the shortest travel times from the following data and the earlier procedure to calculate the expected average response time of an ambulance from our optimal solution.

Figure 8: Average Ambulance Travel Time and Population Per Zone

|  | Average Ambulance Travel Time (minutes) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zone | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| $\mathbf{2}$ | $\underline{8}$ | $\underline{1}$ | $\underline{6}$ | 18 | 16 | 16 |
| $\mathbf{5}$ | 18 | 16 | 10 | $\underline{4}$ | $\underline{2}$ | $\underline{2}$ |
| Population | $\mathbf{5 0 , 0 0 0}$ | $\mathbf{8 0 , 0 0 0}$ | $\mathbf{3 0 , 0 0 0}$ | $\mathbf{5 5 , 0 0 0}$ | $\mathbf{3 5 , 0 0 0}$ | $\mathbf{2 0 , 0 0 0}$ |

Our $\bar{t}$ for the two-ambulance arrangement is 3.67 minutes. Though this is a larger average response time than we found with three ambulances, it is still small enough for the two ambulances to be labeled successful at handling emergencies.

### 3.2.2 Sensitivity Analysis

Because of the real-world variables involved in the time it would take an ambulance to travel to the emergency site, we decided to once-again test the robustness of our solution by increasing the travel times by ten percent.

Figure 9: Increased Shortest Travel Times for Ambulances in Zones 2 and 5

| Zone | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Response <br> Time <br> (minutes) | 8.8 | 1.1 | 6.6 | 4.4 | 2.2 | 2.2 |

After the increase, we see above in Figure 9 that we no longer have a successful solution to respond to all the zones within the eight-minute time limit. The ambulance traveling to zone 1 cannot arrive in time. The new expected value of $\bar{t}$ is 4.03 minutes, meaning that the average citizen can still expect help within eight minutes. Therefore, placing ambulances in zones 2 and 5 would still be ideal. With zone 1 not being covered, there are 50,000 people who will have to wait an approximate extra forty-eight seconds above the eight-minute time limit.

### 3.2.3 Strengths and Weaknesses

Our model predicts that even under conditions that are not semi-perfect, the average citizen in the county can expect to receive aid well before eight minutes. Under semi-perfect conditions, our model is the only combination that successfully can reach any zone in the county within eight minutes.

However, the sensitivity analysis showed how any delay in travelling to zone 1 specifically would cause the average time to increase and the ambulance to fail to arrive within eight minutes. These few seconds can be crucial to the citizen's health.

### 3.3 One Ambulance

### 3.3.1 Determining Optimal Location

If only one ambulance were available for use, there is no location from which all six zones can be reached within eight minutes by one ambulance. Instead, we found find the optimal location for the single ambulance by finding the largest amount of the 270,000 people that can be reached in the shortest amount of time, seen in Figure 10.

Figure 10: People Serviced and Average Service Time for Single Ambulance

| Zone | People Reached <br> Successfully Within <br> 8 Minutes | People Not <br> Reached <br> Successfully | Average Time to <br> Reach Patient <br> (minutes) |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 130,000 | 140,000 | 9.22 |
| $\mathbf{2}$ | 160,000 | 110,000 | 9.37 |
| $\mathbf{3}$ | 85,000 | 185,000 | 11.24 |
| $\mathbf{4}$ | 85,000 | 185,000 | 12.50 |
| $\mathbf{5}$ | 110,000 | 160,000 | 10.42 |
| $\mathbf{6}$ | 85,000 | 185,000 | 11.59 |

Zones 1 and 2 could both be viewed as the optimal location for an ambulance to be placed. Although a single ambulance in zone 1 would take less time on average to reach an emergency, the average difference is only .15 minutes, or 9 seconds, and an ambulance in zone 2 would be able to reach 30,000 more people within eight minutes. We decided zone 2 would be the best zone for a single ambulance to be stationed in because more people were covered within our time limit and the entire population could be reached in nine more seconds.

### 3.3.2 Sensitivity Analysis

As seen in Figure 11 below, a simple ten percent increase causes an ambulance starting in zone 1 or 2 to reach less people. It is clear that zone 2 is a better place for the ambulance to start as it can reach more than twice the number of people within eight minutes, though the average time to reach the entire population is still slightly higher.

Figure 11: People Reached and Average Response Time with Increased Travel Time

| Zone | People Reached <br> Successfully | People Not Reached <br> Successfully | Average Time to <br> Reach Patient <br> (minutes) |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 50,000 | 220,000 | 10.14 |
| $\mathbf{2}$ | 110,000 | 160,000 | 10.31 |
| $\mathbf{3}$ | 85,000 | 185,000 | 12.36 |
| $\mathbf{4}$ | 85,000 | 185,000 | 13.75 |
| $\mathbf{5}$ | 110,000 | 160,000 | 11.46 |
| $\mathbf{6}$ | 85,000 | 185,000 | 12.75 |

### 3.3.3 Strengths and Weaknesses

Our solution covers the maximum amount of people in the eight-minute time limit that a single ambulance can.

However, it is not the fastest average time for the entire county and doesn't cover all the zones in the county in less than eight minutes. For these reasons, we encourage the Emergency Medical Coordinator to only use this solution in dire cases when one ambulance is available for the entire county.

## 4 Catastrophic Events

### 4.1 Multiple Emergencies and Multiple Zones

We addressed catastrophic events in the county by looking at the most likely scenario: two emergencies occurring at once. We determined the minimal time required for two out of the three ambulances in zones 1,2 , and 5 to reach the emergencies.

Figure 12: Minimum Travel Time (minutes) to Deal With Simultaneous Emergencies

|  | Second Emergency Zone |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |  |
|  | $\mathbf{1}$ | 8 | 1 | 6 | 4 | 2 | 2 |  |
|  | $\mathbf{2}$ | 1 | 8 | 8 | 4 | 2 | 2 |  |
|  | $\mathbf{3}$ | 6 | 8 | 10 | 6 | 6 | 6 |  |
|  | $\mathbf{4}$ | 4 | 4 | 6 | 14 | 10 | 14 |  |
|  | $\mathbf{5}$ | 2 | 2 | 6 | 10 | 10 | 10 |  |
|  | $\mathbf{6}$ | 2 | 2 | 6 | 14 | 10 | 16 |  |

Figure 13: Contour Plot of Minimum Response Times to Simultaneous Emergencies


We then plotted this data on a three-dimensional contour graph, as seen in Figure
13. The $x$ - and $y$-axes represent the first and second emergency zones, respectively. The
z-axis displays the minimum amount of time, in minutes, it would take under semiperfect conditions to arrive at both scenes of the emergencies.

We will be able to handle all emergencies that occur in zones 1 or 2 . We can also handle all emergencies in zone 3 unless both calls come from there. However, we are unable to respond with any emergencies within the eight minutes if both calls came from zones 4,5 , or 6 .

Once we assumed that the calls came in at the same time, we found that in seven out of the twenty-one possible situations the ambulances were unsuccessful in responding to both calls within the eight-minute time limit. If we also assume that each person is equally likely to be in an emergency, then we can calculate what the probability of each of those seven situations occurring is. We initially calculated what percentage of the total population each zone consisted of, seen below in Figure 14.

Figure 14: Population Divided by Zone

| Zone | Percentage of Population |
| :---: | :---: |
| 1 | $18.5 \%$ |
| 2 | $29.6 \%$ |
| 3 | $11.1 \%$ |
| 4 | $20.4 \%$ |
| 5 | $13.0 \%$ |
| 6 | $7.4 \%$ |

We then found the probability that each of the seven unsuccessful situations would occur. We determined this by assuming that the probability of an emergency depended solely on the proportion of the population in the respective zone. In essence,
each person was independent of the other and equally likely to face an emergency. We multiplied the population proportions of each zone involved in the situation together.

Figure 15: Probabilities of Unsuccessful Multiple Emergencies Occurring

| Unsuccessful Situation | Probability |
| :---: | :---: |
| Zones 3, 3 | $1.23 \%$ |
| Zones 4, 4 | $4.16 \%$ |
| Zones 4, 5 | $2.65 \%$ |
| Zones 4, 6 | $1.51 \%$ |
| Zones 5, 5 | $1.69 \%$ |
| Zones 5, 6 | $0.96 \%$ |
| Zones 6, 6 | $0.55 \%$ |
| Total | $12.75 \%$ |

If the Emergency Service Coordinator receives two calls at the same time, the total probability that the ambulances will not be able to reach both emergencies within the eight-minute time limit is $12.75 \%$. This also means that the probability that the ambulances will reach both emergencies in time is $100 \%-12.75 \%=87.25 \%$, which shows that our optimal arrangement of the ambulances is also efficient at handling more than one call at a time.

Next, we addressed the question of how to maximize coverage when a catastrophe located in one zone affects people from all zones. It does not matter which zone initially the catastrophe occurs in, because we focused on how to help the victims in all the zones. We used the three ambulances placed in zones 1,2 , and 5 to find the optimal routes for each in case of such an emergency. The ambulances would have to cover, between them, all six zones. We found two different optimal solutions.

Figure 16: Minimum Response Time of Two Different Ambulance Configurations

|  | Zone 1 Ambulance |  | Zone 2 Ambulance |  | Zone 5 Ambulance |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Zones <br> Covered | Total <br> Time <br> (minutes) | Zones <br> Covered | Total <br> Time <br> (minutes) | Zones <br> Covered | Total <br> Time <br> (minutes) | Minimum <br> Response <br> Time |
| $\# 1$ | 1,2 | 9 | 3,6 | 10 | 5,4 | 6 | 10 minutes |
| $\# 2$ | 1 | 1 | $2,3,6$ | 11 | 5,4 | 6 | 11 minutes |

Figure 16, above, shows those two possible solutions and details what routes are taken and how much time is required. In option one, the first ambulance would begin in zone 1 and address problems in zone 1 and then zone 2 . The second ambulance would begin in zone 2 but move straight to zone 3 and address problems in zone 3 and then zone 6. The third ambulance would address problems in zone 5 , where it started, before moving to cover zone 4 . The second ambulance takes ten minutes, making it the longest route of the solution. This is our minimum response time, assuming that as soon as an ambulance reaches a zone the emergency is completely dealt with, since we cannot reasonably estimate how long it would take to help a zone. Option two is similar, with the only difference being that the zone 1 ambulance only covers zone 1 , while the zone 2 ambulance covers zone 2 before moving to zone 3 . This new route for the zone 2 ambulance takes eleven minutes because it requires an extra minute to cover zone 2 . Though both of these minimum response times are over our eight-minute limit, they are still very quick when considering a countywide disaster.

Although the second option requires an extra minute, it covers a larger amount of people within the eight-minute time limit, as shown in the following tables, Figures 17 and 18 , where we determined the number of people successfully reached within eight minutes.

Figure 17: Option One's Coverage Under Eight Minutes

| Zone \# | $\begin{array}{c}\text { Coverage } \\ \text { (minutes) }\end{array}$ | $\begin{array}{c}\text { Under 8 } \\ \text { min? }\end{array}$ | Successfully |
| :---: | :---: | :---: | :---: |
| Covered |  |  |  |
| People |  |  |  |$\}$

Figure 18: Option Two's Coverage Under Eight Minutes

| Zone \# | $\begin{array}{c}\text { Coverage } \\ \text { (minutes) }\end{array}$ | $\begin{array}{c}\text { Under 8 } \\ \text { min? }\end{array}$ | Successfully |
| :---: | :---: | :---: | :---: |
| Covered |  |  |  |
| People |  |  |  |$\}$

Though the people in zones three and six will be reached a minute later, option two still covers 80,000 more people than option one does. It is for this reason that we would suggest the second option as the optimal solution to responding to a catastrophic event.

These figures come from the average response time under semi-perfect conditions. It is unlikely that during a catastrophe of such a large magnitude semi-perfect
driving conditions would be achieved. Also, it is unlikely that an ambulance can simply solve the emergency by reaching the site of the disaster, and there will probably be a large volume of people to treat in one zone before moving onto the next, causing the actual times to be much larger than expected. However, our model does provide the basis for the residents and officials in the county to assign ambulance vehicles to zones based purely on travel times. A higher travel time hurts the victims, because they are not able to seek medical help until the ambulance arrives. Minimizing travel time encourages the quickest medical response.

### 4.2 Planning for an Emergency

If a catastrophic event occurred in one location with many people from all zones involved, there is no way the Emergency Service Coordinator could cover the situation. Even though the three ambulances would be able to reach any of the three zones within the time limit of eight minutes, there is no way for only three ambulances to address all six zones at once, especially when considering the sheer volume of people that would be seeking medical attention at the time. This sudden inflow of emergency response calls is not accounted for in our model, which assumes only one call at a time. Within a single zone, one ambulance would not be sufficient to cover all of the twenty to eighty thousand people in a serious disaster.

Additionally, many of the roads and highways in the area could be shut down after a disaster, especially a flood or an earthquake (1). If these roads were closed down, the ambulance response time could be severely increased because the ambulance might have to take detours to reach its destination, if it is able to reach its destination at all.

In order to maximize survival, the county should institute procedures to carry out in the event of an impending disaster (3). Many of the emergency plans in similar-sized cities and counties include awareness at the individual level (4). Prior to the occurrence of a disaster, individuals are responsible for knowing what to do. For example, they should learn the evacuation routes for the area and know how to navigate them (4). Some cities even send out calls for evacuation ahead of time in the face of disaster.

Emergency management teams from neighboring counties also often coordinate with each other to align their emergency plans with those of other city departments and allied agencies such as the American Red Cross and the Federal Emergency Management Agency (5). These organizations should be able to help provide resources to those in need of them, and other city departments, such as the police force and the fire department should be set up to assist in emergency response if the situation deems it necessary (5).

Volunteers in the community can also be set up beforehand; for large events, places such as Rutherford County utilize bicycles to cover medical treatment as well as the traditional ambulances, because bicycles are quicker to cover larger crowds and transverse rough terrain (2). In a catastrophic event, where likely roads would be closed down and difficult to cross, a resource such as these bicycles could be key to maximizing survival. Additionally, in a large-scale crisis, another important resource would be the survivors of the event, and since the Emergency Management Services would not be able to reach everyone, the survivors should also be able to rely on each other for help (1).

If people from the county stay informed and act according to the previously instituted county plans, assistance from outside cities and organizations both within and outside of the city besides the Emergency Service Coordinator should minimize the city
coverage as much as possible given the current emergency response availability of the county. Other ways to help citizens respond to an emergency situation is to increase their awareness beforehand and invest in more ambulances, better infrastructure, and community volunteer services.

## 5 Conclusion

To assess in which zones ambulances would be best placed, we attempted to maximize the number of people living within zones that were within eight minutes of an ambulance. In situations where more than a single combination of zones was possible, we minimized the average time required to reach an emergency by weighting the time it took to reach the zone by the population of the zone, using the following formula:

$$
\bar{t}=\sum_{n=1}^{6} \frac{t_{n} * p_{n}}{P}
$$

By focusing on minimizing the travel time and maximizing the proportion of the population reached within eight minutes, we were able to come up with an optimal solution for placing three, two, and one ambulance. When three ambulances were available, they were best placed in zones 1,2 , and 5 . If two ambulances were available, the only combination that reached all the zones within the time limit placed ambulances in zones 2 and 5. Having only a single ambulance for a county of 270,000 seemed impractical because it was not possible to reach every single resident within eight minutes. We determined that the ambulance would be best placed in zone 2, though this would leave 110,000 people that could not be reached in the time limit.

In the case of a major disaster, even three ambulances would be impractical to reach the number of people in need. It is not improbable that there would be two emergencies occurring at the same time. We found that out of all possible two-emergency situations based on the population distribution, we would only be able to successfully reach both emergency sites $87.25 \%$ of the time. If a major event occurred that affects people from all of the zones assigning each ambulance to two zones at once would result
in many of the response times being well over eight minutes. Instead we suggest sending the ambulance in zone 1 cover zone 1 , the ambulance in zone 2 to cover zones 2,3 , then 6, and the ambulance in zone 5 to cover zones 5 then 4 . This will successfully cover all of the zones in 11 minutes and reach 250,000 people within the eight-minute time limit.

## 6 Letter to the Emergency Service Coordinator

Dear Emergency Service Coordinator,
To maximize the protection of your citizens, we recommend stationing ambulances in zones 1,2 , and 5 . Doing so would allow your ambulances to reach the entire population within eight minutes. The average response time for ambulances in these zones to service any given individual would be 2.37 minutes, the lowest of any combination of three zones. If one of the ambulances is called on an emergency, we recommend shifting the ambulances to zones 2 and 5, as this is the only combination that covers all parts of the county within eight minutes. If two of the three ambulances are occupied, the remaining ambulance would be best placed in zone 2 . Though a single ambulance in zone one would take less time on average to reach an emergency, the average difference is only .15 minutes, or 9 seconds, and an ambulance in zone two would be able to reach 30,000 more people within eight minutes.

You may find it necessary to send more than one ambulance out at a time. In this case, emergencies in zones 1 and 2 can be dealt with more quickly than in the other zones. In the event of a widespread disaster, three ambulances are very likely too few to adequately respond to the problem due to the possibility of damaged roads and the large number of people that need to be reached by a small number of vehicles. Three ambulances also will probably not contain enough equipment to help thousands of citizens. To ameliorate this deficiency, roads should be kept in excellent conditions, a volunteer bicycle medic force should be set up, other county departments should assist in rescue efforts, and the county should also provide guidelines on how to act during a disaster to its citizens. If there are only three ambulances available, the citizens must be
prepared to wait an extra minute in order to maximize the population that can be reached.
We advise you to deploy ambulances from their locations in zones 1,2 , and 5 to first deal with emergencies in their respective zones. After that, the ambulance in zone 2 should drive to zone 3 and then 6 to assist with emergency response while the ambulance from zone 5 should drive to zone 4 . Following these given recommendations would maximize your offices ability to respond to disasters and help your citizens.

Sincerely,

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