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2013

16th Annual High School Mathematical Contest in Modeling (HiMCM)
Summary Sheet
(Please attach a copy of this page to each copy of your Solution Paper.)
Team Control Number: 4185
Problem Chosen: A

Summary

The goal of our model is to help Emergency Service Coordinator (ESC) maximize the number of residents in the six population zones that can be reached within 8 minutes of an emergency call in a country by determining where the county should station its ambulances.

At first, we determined the best location to place one, two or three ambulances by using the Boolean model, which assigns a “true” or “false” property for an ambulance’s ability to reach a zone in 8 minutes. Then by examining the combinations closely, we obtain the maximum value of population coverage: there were 11 solutions for total coverage with 3 ambulances, a unique solution for total coverage with 2 ambulances, and a maximum of 160,000 people covered with only one. This model is useful due to its zero-tolerance restrictions on the solutions, but cannot differentiate between acceptable solutions.

Second, we improved our model by minimizing transit time, while taking population into consideration. This model uses weighted averages to find the average amount of time a system of ambulances takes to rescue a random patient from any zone, which are more likely to come from populous zones. We obtained the most optimal locations for different situations: a lone ambulance is best stationed in zone 2, a pair of ambulances are best stationed in zones 2 and 5, and three ambulances are best stationed in zones 1, 2, and 5.

Third, we created two models to examine the optimal solution for dealing with catastrophic situations, where many people from different zones make 911 calls. The first method was the nearest neighbor method, in which we create a near-optimal route shared by 3 ambulances, passing through all zones. Then in this route, we placed hospitals at every other point. Using this method, we can on average rescue a person every 2.722 minutes.

The second method, split-cycle method, takes a more complicated approach: it divides the six zones evenly into three areas, with one ambulance responsible for one area. By comparing different combinations produced, we are able to find an optimal solution where ambulances are stationed in zones 2, 4, and 5. This method is both more effective and practical: it averaged only 2.118 minutes to rescue a person.

Emergency Medical Response

2013 HiMCM Problem A

Team #4185

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1.0 Introduction

Throughout history, human beings have long been plagued by diseases and disorders that are often accompanied with unbearable sufferings and serious consequences if they are not met with appropriate treatment promptly. Therefore, countries will often face major challenges to protect their populations from an increasing number of potential health threats. Despite the differences in each country's method, they all share a common goal of maximizing the number of people who can be saved in the shortest amount of time. Consequently, preparedness and prevention have great practical uses and will play a significant role in ensuring an efficient response to a disaster, since a well-designed placement of ambulances and hospitals can be more comprehensive.

For this purpose, Emergency Service Coordinates system forms an integral part of the public health care system, as their function is to deliver emergency medical care in response to emergency calls. The development of Emergency Service Coordinates successfully provides security to residents. Currently, there are 27 EU countries which are implementing the Emergency Service Coordinates[1], and there is continually growing awareness of the importance of emergency medical response. Eight minutes is set as a common standard for medical responses since it is considered both practical and ideal to maximize the survival chances of the patients. Research has shown that for patients with a response time ≥ 8 minutes, 7.1% died, compared with 6.4% for patients with a response time < 8. From this information, we can see that the risk difference is 0.7%, and those who died < 8 minutes is 10% less than those who died ≥ 8 minutes[2]. This also indicates the importance of the concept of time that we will consider in our model.

Our model seeks to mimic the purpose of Emergency Service Coordinates, based on the ability of a defined population to obtain or receive medical aid during an emergency. The main focus is how to maximize the efficiency of the ambulances and cover as many people as possible during an emergency. The most crucial factor is the geological location of the ambulances since it directly determines the effectiveness of medical response, mobility of the ambulances, and the time needed to reach the site (the closer it is, the less time needed). It is also proven in many countries that a well-designed placement of ambulances and hospitals can be more comprehensive.

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[1] "Emergency Medical Service Systems." World Health Organization, 2008. Web. 10 Nov. 2013. <https://ec.europa.eu/digital-agenda/sites/digital-agenda/files/WHO.pdf>.

[2] NCBI. Emergency medical services response time and mortality in an urban setting. Nation Institutes of Health, 25 Oct. 2011. Web. 10 Nov. 2013. <http://www.ncbi.nlm.nih.gov/pubmed/22026820>.

Research has shown that for patients with a response time ≥8 minutes, 7.1% died, compared with 6.4% for patients with a response time < 8. From this information, we can see that the risk difference is 0.7%, and those who died < 8 minutes is 10% less than those who died ≥8 minutes[3]. This also indicates the importance of the concept of time that we will consider in our model. In any cases, the shorter the time the better since there is a higher possibility that the patient will be saved.

1.1 General Assumptions

1. The model takes semi-perfect conditions as uneventful situations and sees the given transit times as the average transit times when there are no unexpected obstacles.

Explanation：Obstacles or accidents are highly unpredictable, and taking them into account not only would complicate our models, but also would not affect the solution, as such accidents can occur no matter what route is taken.

2. Maximizing covered population has the highest priority.

Explanation：Although from a moral perspective, each human life is valued equally, but in case for an emergency, our model must consider how to maximize the number of people who are being covered, if it is indeed impossible to cover the entire population.

3. Ambulances only pick up patients at one location and must drop them off at a hospital before responding to other patients.

Explanation：In real life, at least one ambulance is sent to one affected area, but not many ambulances travel to another patient before returning to the hospital. Taking into account multiple patients would complicate our model and doing so would entail delaying the rescue of the first patient.

4. Prior to receiving emergency calls, the ambulances remain in their hosting hospitals. These hospitals are located in the center of a zone, instead of being between zones.

Explanation：An emergency call cannot be predicted and passively moving from location to location is not always helpful. There also might not be infrastructure to build hospitals in less populated in-between areas.

5. When traveling between different zones, the centers of the two zones are used.

Explanation：We assume the location of a patient in a zone is a distribution within the zone where the average location is the center. That way when traveling between zones, some patients are closer than the zone center while some patients are equally further. But when traveling within a zone, the average distance between patient and hospital is nonzero.

6. When an ambulance reaches a patient, they are considered saved.

Explanation：We assume the ambulances are equipped with adequate medical supply for emergency aid at the site, such that they can begin treatment immediately after pickup. Thus when only one patient is involved, the only transit time that needs to be considered is the time to takes to reach them.

Exception：However, in catastrophic emergencies, the return time is still a significant delay before reaching the next patient, so it is also considered.

7. Only hospitals hosting ambulances can receive ambulances and an ambulance can use any such hospital interchangeably.

Explanation：Our model cannot be made if the number of ambulance-receiving hospitals is unknown, yet in real life emergency scenarios do not call for distinction between hospitals.

8. People will not call for emergency aid in close succession, except in the catastrophic event.

Explanation：In other words, all ambulances are idle when a call is made and only one rescue attempt is made at a time. It is rare to find two patients independently requiring emergency, and in many such cases different ambulances from a 3 ambulance system are to be sent anyway.

Exception：But in the catastrophic scenario people from all zones call nearly simultaneously, resulting in an incessant stream of calls

9. The time needed to prepare an ambulance is not considered.

Explanation：This is because preparation time can be highly variable – whether a previous call occurred recently, whether the call occurred during day or night, and whether the hosting hospital is well staffed. Taking these into consideration is impractical, so we find it acceptable to assume all ambulances are vigilant and prepared for any scenario at all times.

10. The frequency of 911 calls per zone is proportional to the zone's population.

Explanation: We assume that there are little differences between residents of each zone, so the number of potential patients is proportional to the zone's population. The exact characteristics of each zone are not known, nor do their populations differ enough to introduce population-related problems only in certain zones. Thus the distribution of 911 calls should match the population distribution.

2.0 The Models

2.1 Approach

The question asks us whether we can "cover" certain zones or not. Rather than simply looking at the travel times between two zones, however, we applied the Boolean method, which determines if we can reach a certain zone in 8 minutes or not, and outputs a simple yes/no answer. Since we also need to calculate how many, if any, people are left uncovered, we later merged the two graphs to determine the population of zones covered by an ambulance stationed at a certain zone.

Of course, in reality time is an important factor in rescuing patients; therefore, we considered another method which distinguished between different time values rather than just "true/false" conditions, so we created a weighted average method.

The weighted average method calculates the average time spent saving a random patient, and compares combinations of zones to find a position which minimizes this value. Obviously, this method is more informative than the Boolean method, but when it comes to a decision, the Boolean model results take higher priority. This method is useful for differentiating between equally attractive solutions.

2.2 The Boolean Model

Our first model was concerned about whether an ambulance could reach a zone within 8 minutes and "cover it" or not, and our goal was to maximize the number of people "covered". We took a Boolean approach, and assigned values of "T" to a zone if it could be reached in less than 8 minutes; if it couldn't be reached, then "F" was assigned. When multiple ambulances were considered, we merged the "T"s, meaning that a zone is covered as long as one ambulance can reach it within 8 minutes.

Table 1: Boolean Property of Zone 1 to Zone 6

Zones	**1**	**2**	**3**	**4**	**5**	**6**
1	T	T	F	F	F	F
2	T	T	T	F	F	F
3	F	F	T	F	T	T
4	F	F	T	T	F	F
5	F	F	F	T	T	T
6	F	F	T	F	T	T

T: The average travelling time ≤ 8 minutes
F: The average travelling time > 8 minutes

The table shows the results if the only ambulance is placed separately from zone

1 to zone 6. Since the ambulance has to arrive at the site within 8 minutes, any time greater than 8 is dismissed and deemed as “false”, while the time less than or equal to 8 is accepted as “true” and desirable.

The total number of people who can be covered is determined by the sum of the corresponding values of “T”s.

Example working for zone 1

If the ambulance is at zone 1, it can only reach zone 1 and zone 2 within 8 minutes, but not zone 3, zone 4, zone 5, and zone 6. Therefore the sum is calculated by

Population of zone 1 + Population of zone 2 = 50,000 + 80,000 = 130,000

(For zone 2 to zone 6, follow the same procedure.)

Table 2: The population covered by 1 ambulance in Zone 1 to Zone 6

Zone	**Population Covered**
1	130,000
2	160,000
3	85,000
4	85,000
5	110,000
6	85,000

2.2.1 Solution

1. ***Determine the locations for the three ambulances which would maximize the number of people who can be reached within 8 minutes of a 911 call. Can we cover everyone? If not, then how many people are left without coverage?***

When the number of ambulance available = 3, all zones can be covered in multiple ways. Since 2 ambulances are sufficient to solve the problem (shown below), as long as all zones are covered, we no longer distinguish between different methods.

To specify our method, we considered all possible combinations of the 3 ambulances that can reach a total population covered 270,000, and also taking into account the possibility of overlapping.

Conclusion: I. All zones can be covered.
II. There are in total 11 solutions that can result in all zones being covered.

Table 3: 11 Solutions for combination of 3 ambulances covering total population

Zone /Solution	1	2	3	4	5	6
Solution 1	✓	✓			✓	
Solution 2		✓	✓		✓	
Solution 3		✓		✓	✓	
Solution 4		✓			✓	✓
Solution 5	✓		✓	✓		
Solution 6	✓		✓		✓	
Solution 7	✓			✓		✓
Solution 8	✓				✓	✓
Solution 9	✓			✓	✓	
Solution 10		✓	✓	✓		
Solution 11		✓		✓		✓

2. ***We now have only two ambulances since one has been set aside for an emergency call; where should we put them to maximize the number of people who can be reached within the 8 minute window? Can we cover everyone? If not, then how many people are left without coverage?***

When number of ambulance available = 2, the situation becomes more difficult, and only one combination can satisfy the conditions: two ambulances placed in zones 2 and 5. The ambulance in zone 2 covers zones 1, 2, and 3, whereas the one in zone 5 covers zones 4, 5, and 6. Through this combination, all of the population is covered.

The answer is obtained using the similar method of testing combinations and checking if all population zones can be covered.

Conclusion: I. All zones can be covered.
II. There is a unique solution: specifically, two ambulances are positioned at zones 2 and 5, respectively

3. ***Two ambulances are now no longer available; where should the remaining ambulance be posted? Can we cover everyone? If not, then how many people are left without coverage?***

By referring to *Table 2: The population covered by 1 ambulance in Zone 1 to Zone 6*, when the number of ambulance available = 1, the maximum population covered in this case is when the ambulance is at zone 2, which corresponds to a value of 160,000.

The total population is 270,000, but 160,000 < 270,000, therefore it cannot cover everyone.

population left without coverage = total population – population who are covered
= 270,000 – 160,000
= 110,000

Conclusion: I. The remaining ambulance should be posted at zone 2
II. Not everyone can be covered
III. 110,000 people are left without coverage

A brute force program confirms our conclusion:

```
          Zone 1     Zone 2     Zone 3     Zone 4     Zone 5     Zone 6
Uncovered: 0 (1m)     0 (1m)     0 (6m)     0 (4m)     0 (2m)     0 (2m)

Chose 1, 2 and 5. Total uncovered = 0.
Total number of solutions with 3 ambulances = 11, with 2 ambulances = 1,
with 1 ambulance = 0
```

This program shows a solution for 3 ambulances, and the numbers of solutions.

2.2.2 Strengths and Weaknesses (Boolean Model)

Strengths:
1. It is efficient to calculate the results since we only need to consider "true" and "false" by comparing the average travelling with a standard of 8 minutes then find the corresponding values

2. It allows us to completely evaluate all sets or collections and pick the best system with certainty. A perfect solution is readily identifiable.

3. It gives a hard-cut standard and harshly punishes results that cannot fulfill the minimum requirement. Real life situations often allow zero tolerance, especially in situations of life and death, where assistance within 8 minutes is a mandatory standard. This makes the acceptable solutions stand out from the near-misses.

Weaknesses:
1. By only considering "true" and "false", the model neglects the relative importance of time, that is, the shorter the better. In other words, the model is simply concerned with the range of time. It does not differentiate time periods within the same range. For example, although a 3 minute arrival time is more preferred compared to a 6 minute arrival time, they are valued equally in the Boolean Model as "true".

2.3 Weighted Average Model

Transit times (T)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	
Ambulance in Zone 1	1	8	12	14	10	16	
Ambulance in Zone 2	8	1	6	18	16	16	
Ambulance in Zone 3	12	18	1.5	12	6	4	
Ambulance in Zone 4	16	14	4	1	16	12	
Ambulance in Zone 5	18	16	10	4	2	2	
Ambulance in Zone 6	16	18	4	12	2	2	
Population (P)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	
270000	50000	80000	30000	55000	35000	20000	
Contribution to wAvg (C)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	wAvg = Sum
Ambulance in Zone 1	0.185185	2.37037	1.333333	2.851852	1.296296	1.185185	9.222222222
Ambulance in Zone 2	1.481481	0.296296	0.666667	3.666667	2.074074	1.185185	9.37037037
Ambulance in Zone 3	2.222222	5.333333	0.166667	2.444444	0.777778	0.296296	11.24074074
Ambulance in Zone 4	2.962963	4.148148	0.444444	0.203704	2.074074	0.888889	10.72222222
Ambulance in Zone 5	3.333333	4.740741	1.111111	0.814815	0.259259	0.148148	10.40740741
Ambulance in Zone 6	2.962963	5.333333	0.444444	2.444444	0.259259	0.148148	11.59259259
Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037	
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037	

Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148	
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741	
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0	
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0	

This model takes both timing and population under consideration. It uses these tables to help calculate and minimize the weighted average transit time for any system. It is necessary to first explain these tables, before going into the approach

T is the amount of time an ambulance spends to reach a destination.
P is the population of each zone. The first cell shows the total population.
C represents the absolute contribution to the weighted average time for an ambulance at each departure location. It is equal to

$$C(\boldsymbol{i},\boldsymbol{j}) = \frac{P(\boldsymbol{j})}{P(total)} \times T(\boldsymbol{i},\boldsymbol{j})$$

Where ***i*** represents the departure zone, and ***j*** represents the destination zone.

The sum of each zone's contribution is the weighted average time an ambulance spends reaching a destination. The magnitudes of the contributions represent their impact on the weighted average – the higher their contribution, the more they hurt the weighted average time.

An absolute contribution value is used because the final goal is independent of the weighted average time one specific ambulance spends to reach all other zones. As the formula shows, the ambulance's performance in other zones does not affect this value – it is only related to the population and time needed to reach a specific zone. Because of this, these values can also be used on a system that sends different ambulances to different zones. The weighted average transit time of the system is still the sum of each zone's absolute contribution.

The weighted average is used because population is an important consideration when designing emergency health care transportation. Zones with higher population are more likely to call ambulances. A weighted average takes distance and population into consideration: populous, distant zones show higher absolute contribution than less populated, nearby zones. This value represents the average time needed to save a random patient; it is much more likely to come across a patient from populous zones.

ΔC compares the contribution of a destination zone between departure zones. A value of 0 represents the departure-destination pair with the lowest contribution to the weighted average out of all other pairs with the same destination. Each destination zone must have at least 1 departure location most suited for that zone. It is not surprising that the ambulances traveling within their departure zones are the ones that display a $\boldsymbol{\Delta}\mathbf{C}$ of 0.

The remaining values display their difference in contribution from the minimum. A low value represents a departure-destination pair that is a good alternative for the minimum, while a high value represents a pair that is much worse than the minimum. The smallest nonzero cell is $\boldsymbol{\Delta}\mathbf{C}$ (3,6). The **T** table shows that an ambulance traveling from zone 3 to zone 6 only takes 2 minutes more than an ambulance traveling within zone 6 – the smallest positive difference in the table. Transit within zone 5 and 6 is as easy as transit between the two, which is why the $\boldsymbol{\Delta}\mathbf{C}$ for both is 0.

Ideal Weighted Average Process

The overall goal is to find a set of 3 ambulances that when chosen aptly, will result in the lowest weighted average time for the system of 3. The weighted average is the sum of the contributions from each destination zone, and for a system of multiple ambulances, the most suitable departure zone is used for each destination. A perfect system would include an ambulance at each zone so that no ambulance has to leave their zone. The weighted average rescue time for this perfect system is

$$C(\mathbf{1},\mathbf{1}) + C(\mathbf{2},\mathbf{2}) + C(\mathbf{3},\mathbf{3}) + C(\mathbf{4},\mathbf{4}) + C(\mathbf{5},\mathbf{5}) + C(\mathbf{6},\mathbf{6}) = 1.259259 \text{ minutes}$$

which is reasonable because the most populous zones – 2, 4, 1 – all take 1 minute on average for an ambulance to transverse within the zone.

The weighted average transit time for a system of 3 ambulances can be calculated by finding the minimum $\boldsymbol{\Delta}\mathbf{C}$ values among the three departure zones for each destination zone, and adding their sum to the ideal transit time. The system with the lowest weighted average is the best system of 3 in terms of transit time. We created the **cross and minimize process** that can help find the minimal solution:

1. Discard the diagonal line of 0's in $\boldsymbol{\Delta}\mathbf{C}$ table
2. Find the destination zone with the smallest minimum value in their column.
3. Check the row corresponding to the destination zone. If previously marked cells are not already in the row, then mark the cell holding the minimum value, then cross out the entire column and row corresponding to the destination zone.

   3b. Otherwise, compare the next column minimum with the next smallest uncrossed value in a conflicting cell's column. Repeat for each conflicting cell. Move the conflicting cells if the increase in $\boldsymbol{\Delta}\mathbf{C}$ is small, otherwise attempt step 3 on the next column minimum, considering the increase between the minimums.

4. Repeat step 3 three times.

4b. If two departure zones compete for minimum, then separately try both cases.

5. The three ambulances should originate from the rows that haven't been crossed out. The sum of the marked cells should be minimal.

5b. If multiple cases must be considered, the minimal case is the one whose sum of the marked cells is the least

This method uses the assumption that traveling within a zone is no slower than traveling between zones. Thus the ΔC for 3 zones hosting 3 ambulances is automatically 0. The remaining cells are chosen to have the smallest available ΔC. The procedure finds these minimum values, and crosses out columns to help locate the next-smallest minimum; the rows that are crossed out are the 3 zones the ambulance will not depart from (and thus are not automatically 0). The process tries to make sure ambulances depart from zones that are difficult to reach, while they can quickly reach the remaining zones. It assumes that there are enough ambulances to include the best sets of closely linked zones. If step **3b** is never needed, then it will find the optimal solution. **S**tep **3b** simply decides the best changes to make given the current progress if it encounters an error. But if the number of ambulances is too limiting, then step **3b** will be called too often, and not many column minimums will remain.

For questions 1, 2, and 3, we compared a brute force program with the results of this process. This process is more practically feasible when the number of zones to consider increases, because this process runs on polynomial time while the brute force algorithm runs on exponential time. But the brute force algorithm guarantees the ideal solution. We will demonstrate that the above process is very accurate and efficient when the number of ambulances is comparable to the number of zones.

Demonstrating the process

We will demonstrate the cross and minimize process for three ambulances

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

First, the diagonal 0's are removed. The three ambulances chosen should be responsible for their own zone, so three transit times are automatically minimized. Now we are interested in minimizing the transit times to the three other zones.

Next, find the zone with the smallest column minimum. Both zones 5 and 6 have a column minimum of 0, so both will be separately tested. For this example, we select zone 6. Mark the column minimum cell.

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Cross out both the row and the column for zone 6, unless this crosses out a marked cell from a different step. This act means we have decided not to have an ambulance in zone 6, since a different ambulance can reach this location quickly as well. Repeat until a marked cell from a previous step is in the way.

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

The next column minimum is located in zone 3, with a value of 0.277778. We checked the zone 3 row. There are no marked squares from previous steps in this row, so we are given the clear to cross out both the row and column for zone 3. Crossing out a previously marked cell is not allowed because it represents both allowing and disallowing an ambulance to come from the crossed out zone, which is a contradiction.

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

The next column minimum is from zone 4, with a value of 0.611111 (red cell). But crossing out the zone 4 row will cross out the marked cell for zone 3, as the dotted lines show. We have two choices – move the zone 3 cell to the next smallest in the column (the purple cell) or skip zone 4 and go to the next column minimum (green).

Moving involves an increase in value, from 0.5 to 0.2777778, a difference of 0.222222. Skipping involves giving up the current column minimum (0.611111), and instead choosing a new one (1.296296), a 0.685105 increase.

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Moving makes a smaller change, so we choose to move the marked cell. Then there are no longer conflicts in the zone 4 row, so we can finalize it. Sometimes more than one cell has conflicts. Sometimes moving to another zone also introduces conflicts. But in this case there are no more issues. In the end, the purple and the red cells are marked.

With three iterations, the cross and minimize process is now done. The solution is the set of zones that are not crossed out – 1, 2 and 5.

2.3.1 Solutions

Three ambulances

We ran the whole process on the two possible starting positions – zone 5 or zone 6. We then compared these two to find the starting position that yields a shorter time.

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Sum of marked squares = 1.574074. Uncrossed rows = 2, 4, 6
Zone 6 skipped for zone 1. Less costly than moving cell in zone 5.

Zone 6 crossed out first:

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Sum of marked squares = 1.111111. Uncrossed rows = 1, 2, 5
Cell was moved to cross out zone 4. Less costly than skipping to zone 1.

The second approach gave a smaller result, so the best combination of ambulance departure zones will be **Zone 1, Zone 2, and Zone 5.**

To find the weighted average transit time of the system, recall that weighted average is the sum of the absolute contributions from the 6 zones, and that**ΔC** is the difference between the absolute contribution and the column minimum.

Thus the weighted average is the sum of the 6**ΔC**’s and the 6 column minimums.

The sum of the column minimums is 1.259259 minutes, while the sum of the **ΔC**’s is equal to the three marked squares (as well as 3 automatic 0’s). Thus

$$Weighted\ Average = (1.259259 + 1.111111)\text{ minutes} = 2.370370\text{ minutes}$$

The brute force algorithm tried all combinations of three. It outputs

```
3 Ambulances:
The best departure zones are 1, 2 and 5; the combination's weighted average is 2.370370.
```

Thus confirming the accuracy of the process

<u>**Two ambulances**</u>

The same process on the **ΔC** graph can be used, except the third step must be done 4 times. Again, zone 5 and 6 compete for minimum.

Zone 5 crossed out first:

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Sum of marked squares = 4.129630. Uncrossed rows = 2, 4

Zone 6 skipped, used zone 1. Less costly than moving cell in zone 5.

Cell was moved to cross out zone 6. Skipping to zone 4 or 2 involves costly moving of cells in zones 3/5/6 or in zone 1.

Zone 6 crossed out first:

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Sum of marked squares = 2.407407. Uncrossed rows = 2, 5

Cell was moved to cross out zone 4. Less costly than skipping to zone 1.

Skipped zone 5 for zone 1. Less costly than moving cells in zone 4 and 6.

The two ambulances should depart from zones 2 and 5.

Weighted average:

$$Weighted\ Average = (1.259259 + 2.407407)\text{ minutes} = 3.666667\text{ minutes}$$

The brute force program outputs

```
2 Ambulances:
The best departure zones are 2 and 5; the combination's weighted average is 3.666667.
```

Again consistent with the output of the cross and minimize process.

One ambulance

Again, the same process on the **ΔC** graph can be used, except the third step must be done 5 times. Only 1 zone will remain uncrossed.

Zone 5 crossed out first:

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0
Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Sum of marked squares = 8.111111. Uncrossed row = 2

Zone 6 skipped, used zone 1. Less costly than moving cell in zone 5.

Cell was moved to cross out zone 6. Skipping to zone 4 or 2 involves costly moving of cells in zones 3/5/6 or in zone 1.

Cells were moved to cross out zone 4. Skipping to zone 2 involves costly moving of cells in zone 1.

Zone 6 crossed out first:

Deviation from min C (ΔC)	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Ambulance in Zone 1	0	2.074074	1.166667	2.648148	1.037037	1.037037
Ambulance in Zone 2	1.296296	0	0.5	3.462963	1.814815	1.037037
Ambulance in Zone 3	2.037037	5.037037	0	2.240741	0.518519	0.148148
Ambulance in Zone 4	2.777778	3.851852	0.277778	0	1.814815	0.740741
Ambulance in Zone 5	3.148148	4.444444	0.944444	0.611111	0	0

Ambulance in Zone 6	2.777778	5.037037	0.277778	2.240741	0	0

Sum of marked squares = 8.111111. Uncrossed row = 2

Cell was moved to cross out zone 4. Less costly than skipping to zone 1.

Skipped zone 5 for zone 1. Less costly than moving cells in zone 4 and 6.

Cells were moved to cross out zone 5. Skipping to zone 2 involves costly moving of cells in zones 1/3.

The brute force algorithm simply compared the weighted average of each row

```
1 Ambulance:
The best departure zone is 1; its weighted average time is 9.222222.
```

A direct comparison of the weighted averages states that the best departure zone is **not** zone 2. As the diagram shows, the cross and minimize process is not very practical on 1 ambulance. There were many cell swaps, and the column minimums were skipped or moved many times. Zone 2 was the second-best choice, though, at 9.370370 minutes.

2.3.2 Strengths and Weaknesses (Weighted Average Model)

Strengths:

1. It considers the relative importance of both time and population, weighing both factors into calculation. The weighted average is a very realistic and important value, representing the average amount of time it takes to respond to a 911 call in general.

2. The process has a good chance of finding the optimal answer, being especially accurate where the number of ambulances is close to the number of zones. Doing a brute force test on such conditions can take much more processing power.

Weaknesses:

1. The calculation process is rather intricate and complicated. This method is less efficient than the Boolean model, while obtaining similar results.

2. The result of the cross and minimize process is less reliable when the number of ambulances is much less than the number of zones – conditions where brute force takes less computing power.

2.4 General Conclusion

I. Using the weighted-average model, we are able to obtain a much more accurate optimal solution for all solutions: for 3 ambulances, the best solution is to place ambulances at zones 1, 2, and 5; for 2 ambulances, the optimal position is zones 2 and 5; for one ambulance, the best position is at zone 2.

II. For each ambulance removed, the average weighted time increased disproportionally: when there were three ambulances, the weighted average was 2.370 minutes; when one ambulance was removed, the time only increased to 3.667 minutes, yet when another is removed, the time increased drastically to 9.222 minutes.

2.5 Summary

According to our assumptions, we prioritize covering the whole population. The Boolean model offers a hard cut pass-fail test for any combination of 3 ambulances. According to this model, 11 combinations of 3 ambulances and 1 combination of 2 ambulances are capable of accomplishing this. A lone ambulance should be stationed in zone 2. Only if a combination passes the Boolean test will it be eligible for analysis from the weighted Average Model, which seeks to minimize the average time taken to rescue a patient. The solution for 3 and for 2 ambulances is included within the Boolean solution set, proving they are best combinations. The one ambulance solution was zone 1, which is more efficient but covers less population. Thus we chose to stick with zone 2 instead, prioritizing population over transit time.

3.0 Catastrophic event

4. (1) If a catastrophic event occurs in one location with many people from all zones involved, could the ESC cover the situation?

3.1 Approach

Every ambulance has a duty to pick up patients and bring them to a hospital as soon as possible. Thus they must alternate between hospitals and patients. Our goal is to find the best routes in case multiple ambulances must be dispatched simultaneously to respond to an emergency situation.

If the ESC has 3 ambulances available, it's best to always run 3 rescue operations simultaneously. To ensure that every zone has an ambulance nearby, the three ambulances should ideally be placed far from each other. If two ambulances operate in the same or in nearby zones, then their coverage areas will overlap with each other, and their efficiency will be reduced. Thus after reaching an uncovered zone, all three ambulances may choose to either return to their original stations or may each proceed to a different one.

Under these criteria, there are two main approaches to designing these routes: designing a circuitous route through all zones, or partitioning the county into three independent areas. We used to Nearest Neighbor Model to tackle the former approach, and the Split Cycle Model to approach the latter.

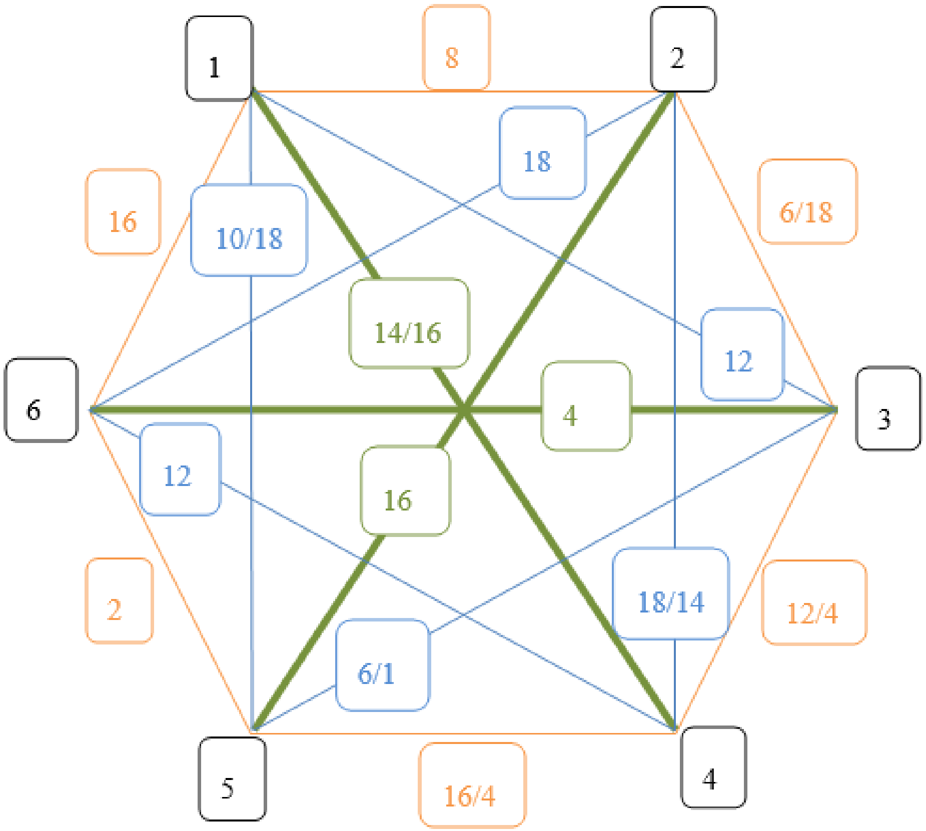
3.2 The Nearest Neighbor Model

When a catastrophic event occurs, we must guarantee that each zone is covered by multiple ambulances, to cope with a situation where several people from different zones require medical aid. Thus, we thought of using the nearest-neighbor model to create a circuitous route, a fairly decent approximation of the optimal solution.

The nearest-neighbor approach basically chooses the immediate minimum cost choice without considering long-term consequences. A path is laid from nearest neighbor to nearest neighbor until all zones are visited; then it returns to the beginning point. Although this may not produce the best result, it has the advantage of being easy to calculate, so we can experiment with the method multiple times and choose the best starting point and route. It is not rare for this path to be the most optimal path, especially in a situation where only 6 points are involved.

Since "catastrophic events" are emphasized, we assume that: 1) multiple people from multiple zones require assistance; 2) ambulances are allowed to stop at different hospitals ("bases") to drop off patients.

Below is a graph of the distribution of the six zones



In this graph, the values represent the transit times between zones. Some routes have two numbers written; the first is the time taken to from the lesser numbered zone to the greater numbered zone, and second is the time taken for the opposite route. Between zones 1 and 5, 10 represents the time from zone 1 to 5, and 18 represents the same from zone 5 to 1. This graph helps see the relative proximity between zones.

In an emergency situation, an ambulance stationed in a zone (zone 1, for instance) has two options: to save patients in nearby zones first, or to save distant patients in need first. There are also various options considering which route to take in order to ensure maximum efficiency. To find these routes, we may apply methods used to solve the travelling salesperson problem, such as the nearest-neighbor method.

The nearest-neighbor method is used for approximating the minimum length of a Hamiltonian cycle (a route which goes through all points in a graph and returns to the starting point). Basically, it operates on a greedy basis, taking the shortest path possible without considering its consequences.

We take Zone 1 as an example. If we start at Zone 1, the closest other zone is Zone 2, 8 minutes away. We then go to Zone 2, and find that Zone 3 is closest one (which we haven't been to yet) from there. At Zone 3, Zone 6 is closest—and so on, until we return to Zone 1. In this way, we are able to complete a Hamiltonian cycle starting from Zone 1, and we can calculate that its length is 40 (minutes). Likewise, all the Hamiltonian cycles starting from points 2~6 can be calculated.

3.2.1 Solution

These sets of tables show the length of all the Hamiltonian cycles.

Average Travel Time (mins)						
Zone	**1**	**2**	**3**	**4**	**5**	**6**
1	1	8	12	14	10	16
2	8	1	6	18	16	16
3	12	18	1.5	12	6	4
4	16	14	4	1	16	12
5	18	16	10	4	2	2
6	16	18	4	12	2	2

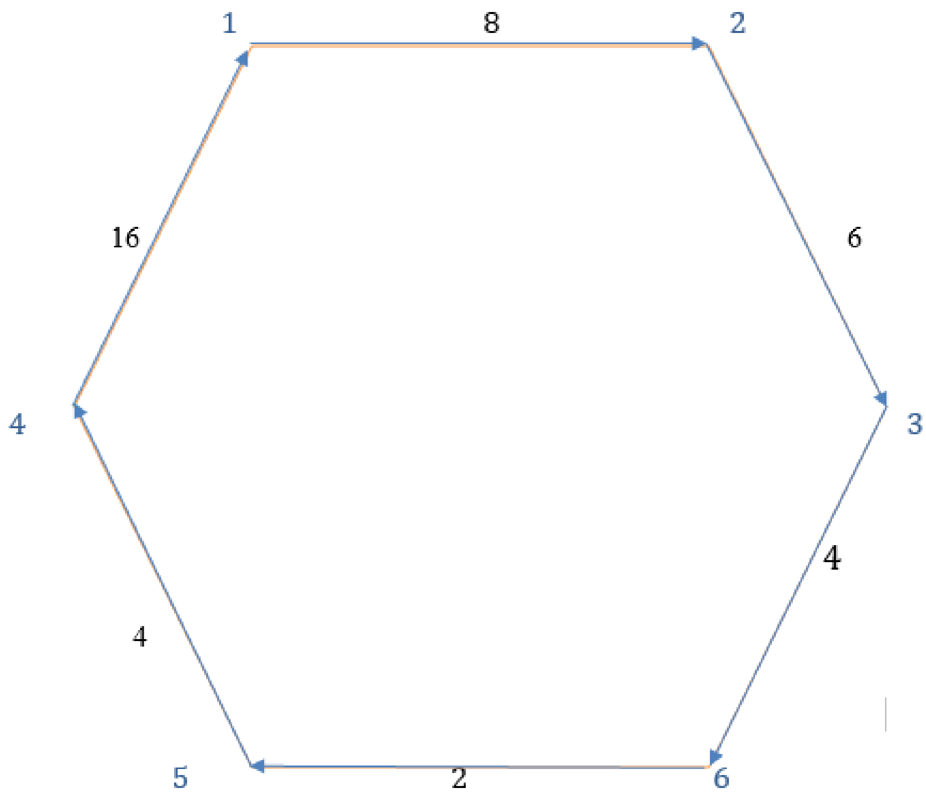
Starting Zone/Step	**1**	**2**	**3**	**4**	**5**	**6**
Start in 1	Go to 2	Go to 3	Go to 6	Go to 5	Go to 4	Go to 1
Start in 2	Go to 3	Go to 6	Go to 5	Go to 4	Go to 1	Go to 2
Start in 3	Go to 6	Go to 5	Go to 4	Go to 1	Go to 2	Go to 3
Start in 4	Go to 3	Go to 6	Go to 5	Go to 2	Go to 1	Go to 4
Start in 5	Go to 6	Go to 3	Go to 1	Go to 2	Go to 4	Go to 5
-	-	-	Go to 4	Go to 2	Go to 1	Go to 5
Start in 6	Go to 5	Go to 4	Go to 3	Go to 1	Go to 2	Go to 6

Zone /Step	**1**	**2**	**3**	**4**	**5**	**6**	**Total (Mins)**
1	8	6	4	2	4	16	**40**
2	6	4	2	4	18	8	42
3	4	2	4	14	8	12	44
4	4	4	2	16	8	14	48
5	2	4	12	8	18	16	60
6	2	4	4	12	8	16	46

It seems the fastest Hamiltonian cycle starts at Zone 1, goes along 1-2-3-6-5-4-1, and takes 40 minutes to complete a roundabout.

However, we must also remember that to complete a cycle and return back to its base, an ambulance must follow a hospital-patient…-hospital route. Since the shortest cycle is 40, as shown above, we may position hospitals as to allow this cycle to form; specifically, position hospitals at 1, 3, and 5 as to allow ambulances to pick up patients between hospitals.

Now, since we have positioned our first ambulance (and hospital), the locations of all three hospitals are now known – Zones 1, 3 and 5. This corresponds to the positions of the three ambulances; we then only need to decide their routes, which is quite simple; given that they are all on the shortest Hamiltonian cycle, the ambulances only need to follow it. For example, if an ambulance has picked up someone at Zone 2, it can follow the path shown below to the next hospital at Zone 3.



However, we cannot ignore the patients from Zones 1, 3 or 5.

At full capacity, the three ambulances travel 40 minutes through the entire circuit, but they also spend an additional 9 minutes rescuing patients next to the hospitals. In those 49 minutes, the ambulances save 18 patients from all 6 zones, for an average of 2.7222 minutes per patient.

Since there are catastrophic situations, however, some ambulances may be forced to take a detour to rescue other patients. When this occurs, we should locate the closest hospital to it (requiring least time to get from hospital to patient), find the ambulance closest *to* that hospital, and make a detour to rescue this patient. For instance, if there is a patient in Zone 4 requiring assistance, we can locate the nearest hospital, which is at Zone 5, and let the nearest ambulance with patients nearby to drop them off at Zone 5, and go to assist the patient in Zone 4. Of course, if there are ambulances with no patients onboard, they may assist the patient in Zone 4, and drop him off at the closest hospital *from* the patient (which is at Zone 3). Throughout the process, we are using the nearest-neighbor method to determine who goes and where to go.

3.2.2 Strengths and Weaknesses (Nearest Neighbor Model)

Strengths:

1. The model was easy to calculate and gave us a "best route" with specific directions, therefore determining the suitable positions for the ambulances (and hospitals).

2. The route is open, allowing entry and exit from any point, which is helpful when ambulances receive emergency calls while running the route.

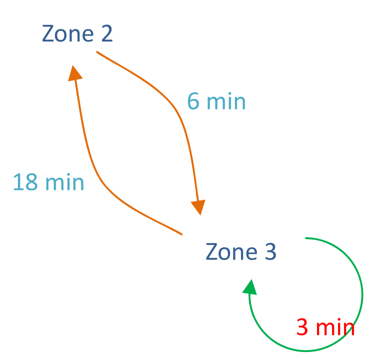
Weaknesses:

1. The model is only an approximation of the optimal solution; to get the best answer still requires more complex methods. Consequently, in some situations the ambulances may not be running at maximum efficiency, taking more time than needed

3.3 The Split Cycle Model

The nearest-neighbor approach attempts to find a quick, universal path for all three ambulances in the form of one closed loop. However, the Split-Cycle method finds 3 small routes, each traveling between 2 of the 6 zones. The 3 routes are independent of one another, so the ease or difficulty of transportation in one zone will not hamper the rescue attempts of an unrelated cycle, whereas every zone affects the time needed to complete a Hamiltonian cycle.

Each of the 3 cycles operates between two zones, one of which hosts a hospital. Ambulances leave their hospital to rescue patients in both zones. If the ambulances save patients from each of the two zones alternatively to save an equal number of patients in each, it will move in a cycle. The amount of time it takes to complete one cycle is equal to double the amount of time taken to move within the hospital's zone, plus the time needed to move back and forth between the two zones. For example, if a hospital was established in zone 3, with the intent to rescue patients in both zone 2 and 3, then under emergency situations an ambulance can rescue 2 patients in 27 minutes.



Whereas each ambulance in the Nearest-Neighbor path moves to a different hospital, the ambulances in this cycle return to their original hospital. As long as every pair of zones contains one hospital, the three cycles can work independently. However, no two cycles can both include the same zone, otherwise some zones will be excluded.

To maximize the efficiency of the Split-Cycle method, the three cycles must have the smallest period. The above case is inefficient because of the slow transit time when traveling from zone 3 to 2.

3.3.1 Solution

We made a program that automated the calculation process between all 6 points, and output the length of one period in a table:

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Zone 1		18	26	32	30	34
Zone 2	18		26	34	34	36
Zone 3	27	27		19	19	11
Zone 4	32	34	18		22	26
Zone 5	32	36	20	24		8
Zone 6	36	38	12	28	8	

Only results between differing zones are displayed, explaining the blank spots.

When one pair of zones is chosen, all other pairs that contain either of the two zones are no longer available. It is not possible to go around this limitation – choosing one point necessarily prevents choosing other points, and vice versa. The order also does not matter, because unrelated pairs never eliminate one another. Thus the best method to pick the optimal pairs is to simply pick the smallest value available, and accept the fact that other favorable pairs may be eliminated. Selecting the pairs like thus, the 3 pairs chosen are (5,6), (4,3), and (2,1). Their periods are 8, 18, and 18 respectively. Note that (5,6) is identical to (6,5), and (2,1) is identical to (1,2). The first in the pair represents the location of the hospital, and the second represents the other zone the hospital is responsible for. The table shows that the hospitals can be established on zones 1/2, zone 4, and zone 5/6.

In an emergency scenario, the 3 cycles work constantly and independently. The periods sync with one another every 72 minutes, where the 8 minute cycle completes 9 times and the 18 minute cycles complete 4 times each, a total of 17 cycles. Each cycle rescues 2 patients, so in the catastrophic scenario, 34 patients are rescued every 72 minutes for an average of 2.118 minutes per patient.

While the table shows that a hospital can be set in either zones 1 or 2, and in either zones 5 or 6, it is possible to find the better of the two. When more and more patients are rescued, eventually all the patients in one zone will be taken to hospitals and the zone is no longer in emergency. The number of patients is likely correlated to population, yet the Split-Cycle saves the same number of patients in all zones. There are two areas where this is significant:

It is very possible that the ambulance in charge of the 8 minute cycle finishes its job before any of the other two. In this case, the idle ambulance can participate in one of the 18 minute cycles. It is possible for two instances of the cycle to run simultaneously, assuming that the local hospital in the cycle can cater to the doubled demand. Zone 4 is an attractive choice if the ambulance originates from zone 5, while zone 3 is equally attractive for ambulances from zone 6; both 4 pairs are minutes apart. But zone 4 is more populous than zone 3, so normally more patients are still awaiting rescue in the former. Thus the hospital in the 8 minute cycle should be built in zone 5.

Similarly, in the zone 2 – zone 1 cycle, there should be more patients in zone 2. An ambulance in zone 2 can reach these patients in 1 minute, while a similar ambulance in zone 1 needs 8 minutes. Thus the hospital should be built in zone 2 rather than zone 1

Considering the table and the ease of assistance from zone 5, the three hospitals should be built in zones 2, 4 and 5, taking responsibility for their own zones as well as zones 1, 3, and 6 respectively. The ambulance operating in zone 5 should assist zone 4 when possible. At highest capacity, a patient can be saved every 2.118 minutes.

3.3.2 Strength and Weaknesses

Strengths:

1. The method uses the optimal 3 pairs, and considers each ambulance to be “assigned” to 2 zones, without interdependence on other ambulances, thereby better utilizing the advantage of having 3 ambulances over 6 zones.

2. The plan is simple to execute, effective, and treats each zone equally. Moreover, difficulties or delays in one cycle do not affect the other two, and when a cycle is prematurely complete, it is still capable of assisting the other cycles. Therefore, it is likely to be applied in real life.

Weaknesses:

1. If one area is affected more severely and requires more ambulances to help, ambulances may have difficulty reaching there. Thus there may be latency before a zone receives additional assistance. After assistance, there is another latent period before the extra ambulances return to their stations, where they cannot respond to other patients.

3.4 General Conclusion and Summary

I. Although the nearest-neighbor method requires less calculation and construction, it gives a less efficient result, on average requiring 2.72 minutes to save one person. In addition, all three ambulances are engaged in the same route but with different bases. This requires ambulances to maintain equilibrium with one another and to be as far from each other as possible in the loop, to ensure maximum efficiency and minimize the time required for patients to receive medical aid. Therefore, applying this method requires synchronization and coordination between ambulances.

II. The split cycle method requires programming and extensive design to derive an answer, but not only is it more efficient (average 2.12 minutes to save one person), it is also easier to follow. Ambulances are assigned two zones each—one with a hospital, and one without. The ambulance then follows this cycle: hospital-nearby patient-hospital-far patient. Thus, the 3 ambulances work separately and do not require coordination with each other. However, if one area has many patients waiting for an ambulance, the two ambulances require more time to respond and need to make major changes to their routes. Overall, it is an easier and more effective way of putting ambulances into use.

3.5 Real World Situation

4. (2) How do counties or cities design for those rare but catastrophic events?

According to National Response Framework (NRF), a catastrophic incident is defined as any natural or manmade incident, including terrorism that results in extraordinary levels of mass casualties, damage or disruption severely affecting the population, infrastructure, environment, economy, national morale, and/or government functions[4]. In general, the immediate response to a catastrophic event is on a local level, by the local government/state government. However, when the need of medical support exceeds the ability of local level and the local governments are unable to sustain a dependable structure of response system, federal assistance can augment medical supplies and can accelerate recuperation. In the case of the United States of America, FEMA (Federal Emergency Management Agency) plays the role in preparing for, preventing, and minimizing the effects of catastrophic incidents, which also makes contracts with other minor agencies to prevent, respond, and recover the situation as effectively as possible. For example, when Hurricane Dean hit the south of Texas in August of 2007, with limited medical supplies on the local level, the state officials of Texas requested Federal assistance with evacuation. Thus, the government

[4] Catastrophic Incident Annex. Department of Homeland Secuirty, Nov. 2008. Web. 10 Nov. 2013. <http://www.fema.gov/pdf/emergency/nrf/nrf_CatastrophicIncidentAnnex.pdf>.

asked AMR (American Medical Response) to provide immediate help with more medical supply. AMR deployed 300 ground ambulances, aircraft and paratransit vehicles from 30 states[5] to handle this catastrophic event and soon it was taken care of.

Other than FEMA, there are still various agencies, which take care of different jobs that help, respond and support any catastrophic incident presented.

Mass Care	DHS/FEMA
Mass Evacuation	DHS/FEMA
Search and Rescue	DHS/FEMA, DOD, U.S. Coast Guard, Department of the interior
Decontamination	DHS, EPA
Public Health and Medical Support:	HHS
Medical Equipment and Supplies	HHS
Patient Movement	HHS, DOD
Mass Fatality	HHS
Housing	DHS, HUD
Public and Incident Communications	DHS
Transportation	DOT
Private-Sector Support	DHS
Logistics	DHS
CIKR Support	DHS

DOD: Department of Defense
DHS: Department of Homeland Security
EPA: Environmental Protection Agency
HHS: Department of Health and Human Services
HUD: Department of Housing and Urban Development
DOT: Department of Transportation[6]

FEMA also encourage precautious preparations by building on and expanding existing organizations and by creating stronger ones, which will contribute to a more stabilized emergency management community. To prevent catastrophic events, the

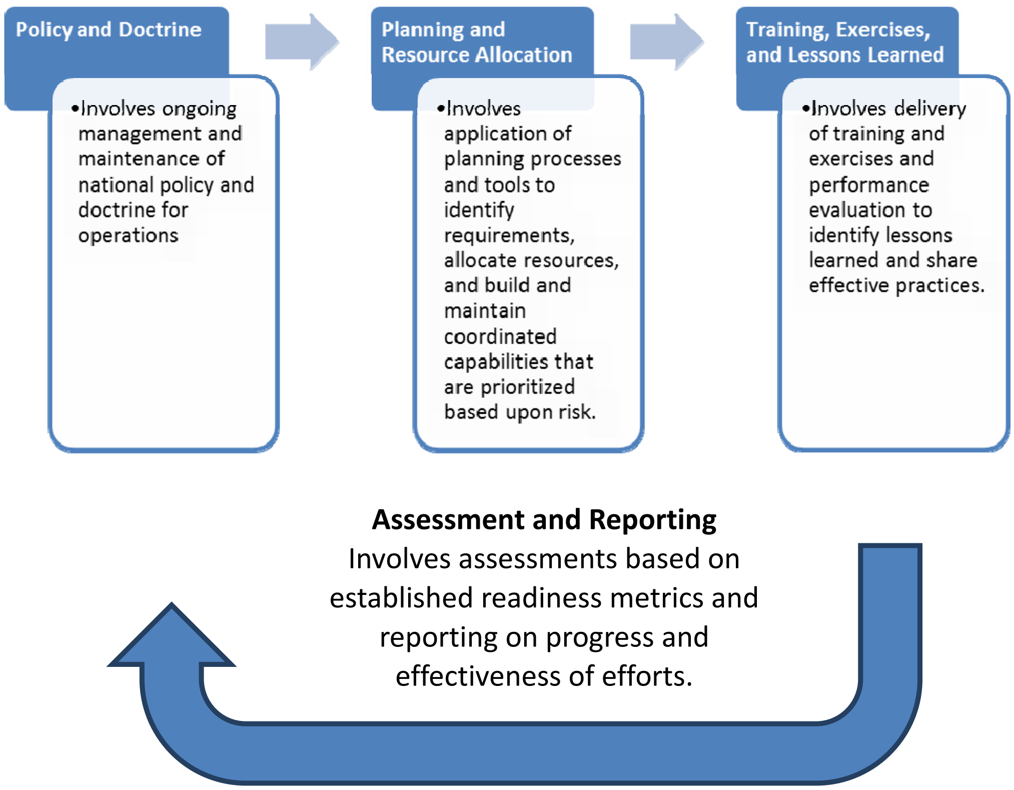
[5] "FEMA Federal Emergency Medical Services Contract." *Overview of AMR*. Federal Emergency Medical Services, 28 Oct. 2013. Web. 10 Nov. 2013. <http://www.amr.net/Files/PDFs/DRT-Companies/AMR-FEMA-contract-overview.aspx>.

[6] Catastrophic Incident Annex. Department of Homeland Secuirty, Nov. 2008. Web. 10 Nov. 2013. <http://www.fema.gov/pdf/emergency/nrf/nrf_CatastrophicIncidentAnnex.pdf>.

government also fosters precautions, discourages manmade disasters, implements awareness through education/propagandas, and takes precautionary measures.

Other actions regard network of communication. The government ensures efficient communication between different departments to avoid overlapping of resource allocation, which is also emphasized in our model.

Another main system is called The National Preparedness System, which is responsible for organizing preparedness activities and programs. The desired end-state of the National Preparedness System is to achieve and sustain coordinated capabilities to prevent, protect against, respond to, and recover from all hazards in a way that balances risk with resources. [7]



[7] *National Preparedness Guidelines*. Home Land Security, Sept. 2007. Web. 10 Nov. 2013. <http://www.fema.gov/pdf/emergency/nrf/National_Preparedness_Guidelines.pdf>.

4.0 Sensitivity Notes

4.1 Sensitivity of the Boolean Model

The Boolean Model has two extreme sensitivity values. It has an extremely high sensitivity when there is a change in average travelling time that crosses the threshold of 8 minutes. For example, if 3 ambulances are used and the average travelling time from zone 2 to zone 1 is changed from 8 minutes to 8.1 minutes, there suddenly will be 5 fewer solutions. However, if the change in travelling time does not cross the threshold, then this model will have zero sensitivity. For example, when 3 ambulances are used and the average travelling time between two certain zones changes from 10 minutes to 100 minutes, the number of solutions remains the same. Therefore, we decided to forgo the sensitivity test, as the lack of sensitivity when not crossing the threshold and the infinitely high sensitivity when crossing it rendered all tests as uninformative.

4.2 Sensitivity test of weighted average model

There are two sets of variables that can be changed: transit time and population. Each variable has a different effect on the model, but overall it takes great and numerous changes to significantly hurt the results.

4.2.1 Transit time

We first made a large change on the transit time from zone 3 to 5, a random entry.

`Transit time from zone 3 to zone 5 changed from 6 → 20`

```
1 Ambulance:
The best departure zone is 1; its weighted average time is 9.222222.
2 Ambulances:
The best departure zones are 2 and 5; the combination's weighted average is
3.666667.
3 Ambulances:
The best departure zones are 1, 2 and 5; the combination's weighted average is
2.370370.
```

There was absolutely no change. This follows our expectations because the cell we edited corresponded to an ambulance departure zone that was already excluded from the solutions. The optimal solutions did not need ambulances departing from zone 3 because zone 3 already performed more poorly than other zones. Making the performance of this cell any worse won’t change anything.

Then we changed the transit time between zone 1 and the relatively distant zone 4.

```
Transit time from zone 1 to zone 4 changed from 14 → 19
```

```
1 Ambulance:
The best departure zone is 2; its weighted average time is 9.370370.
2 Ambulances:
The best departure zones are 2 and 5; the combination's weighted average is 3.666667.
3 Ambulances:
The best departure zones are 1, 2 and 5; the combination's weighted average is 2.370370.
```

The optimal solution for 1 ambulance changed, but none of the others changed. We expected there to be little change, because in a system, only the most suited departure zones are used for each destination. However, in the 1 ambulance case there is no other departure zone that can make up for a slow transit time, so the program decided to switch to departure from zone 2 instead. Because each zone can take the place of others, one would need to change the transit times for multiple cells in order to change the optimal solution by a large amount.

Next we slightly decreased the time needed to move from zone 2 to 3

```
Transit time from zone 2 to 3 changed from 6 → 4.5
```

```
1 Ambulance:
The best departure zone is 2; its weighted average time is 9.203704.
2 Ambulances:
The best departure zones are 2 and 5; the combination's weighted average is 3.500000.
3 Ambulances:
The best departure zones are 1, 2 and 5; the combination's weighted average is 2.203704.
```

The weighted average for all three fell. This decrease was enough to make zone 2 the most optimal departure location for one ambulance. This is because zone 2's relatively short transit time to reach zone 3 is better than the same value for most other ambulances, causing the weighted average to be sensitive to this value. We call this value an important value.

4.2.2 Population

We brought the population of zone 3 very high to see how it would become very significant to the solutions sets and change the average time results

Population of zone 3 changed from 30000 → 300000

```
1 Ambulance:
The best departure zone is 3; its weighted average time is 6.370370.
2 Ambulances:
The best departure zones are 2 and 3; the combination's weighted average is
3.481481.
3 Ambulances:
The best departure zones are 2, 3 and 5; the combination's weighted average is
2.333333.
```

Now the weighted average of all three solutions has fallen, and each solution includes zone 3. This is because much of the population now lives in zone 3. Many emergencies occur there, thus more ambulances pick up patients there. It would be most beneficial to travel within zone 3 to pick up these patients, which takes 1.5 minutes. Thus these solutions all include zone 3, and the average times will fall until it reaches 1.5 minutes at the extreme case with every emergency occurring in zone 3.

Obviously, if one zone has a high population, then it will be important. However, we increased the population of zone 3 to match zone 2 in order to see how the two compare when their population is the same.

Population of zone 3 changed from 30000 → 80000

```
1 Ambulance:
The best departure zone is 2; its weighted average time is 8.843750.
2 Ambulances:
The best departure zones are 2 and 5; the combination's weighted average is
4.031250.
3 Ambulances:
The best departure zones are 2, 3 and 5; the combination's weighted average is
2.906250.
```

Despite their equality in population, zone 2 still seems to have greater importance in the solution set. No overall trend occurs in the average times because the values change for different reasons. Since zone 3 is more important now, the comparatively small distance from zone 2 to zone 3 starts to become visible in the 1 ambulance solution. However, zone 3 remains harder to reach than other zones, so in a combination of ambulances, the increased importance of zone 3 hurts the averages.

We know that zone 1 plays an important role in the optimal solution. We made the population of zone 1 equal to zone 3 to see what role population plays in this fact.

Population of zone 1 changed from 50000 → 30000

```
1 Ambulance:
The best departure zone is 2; its weighted average time is 9.480000.
2 Ambulances:
The best departure zones are 2 and 5; the combination's weighted average is
3.320000.
3 Ambulances:
The best departure zones are 2, 4 and 5; the combination's weighted average is
2.420000.
```

Zone 3 is an average-sized zone, and when zone 1's population is reduced to zone 3's population, suddenly no solutions used it. Instead they preferred zone 4 because it is the next best choice after zones 2, 5, and the former zone 1. The changes in the optimal times shift a bit due to differences between solution sets, but again there is no overall increase or decrease.

4.3 Overall remarks

The three tests on transit times show that this model's sensitivity to transit time depends more heavily on the importance of the changes and availability of close alternatives, rather than the magnitude of the change. In the first test, we increased the transit time of a cell by a large amount, and found no change in the solution. In the third test, a small decrease to one cell decreased all weighted average values.

In general, when a transit time value loses its importance in minimizing the shortest route, any further changes to it will no longer affect the optimal solution. The threshold beyond which a different solution becomes more viable depends on the similarity between rivaling routes – if close alternatives are available, the model is more sensitive to the change. These two factors determine the model's sensitivity to transit time, but changing one value can only change the results so much.

To analyze the role of population, we considered these three factors that contribute to the importance of a zone in minimizing weighted average:

1. The average proximity of the zone to other zones
2. The population of the zone itself
3. The populations of the zone's closest and furthest neighbors

We can use a table of average transit time from each zone to all others to quantify the geographical proximity score for each zone.

Average Time/Zone	Average time to reach other zones (mins)
Zone 1	10.16666667
Zone 2	10.83333333
Zone 3	8.916666667
Zone 4	10.5
Zone 5	8.666666667
Zone 6	9

The sensitivity of the model to population is determined by the complicated interaction between these three factors. A change in population may cause multiple factors to constructively or destructively influence the solution. We can explain the interactions within the 3 population sensitivity tests, and how they determine the sensitivity of the weighted average model.

In the first test, all optimal solutions times fall and tend to 1.5 as population rises. Yet in the second test the average times for multiple ambulances rise, and the times for 1 ambulance falls. Both are effects of the second factor. When zone 3 becomes more important, solutions that include zone 3 become more favorable due to the short transit time for travel within zone 3. Solutions that exclude zone 3 could become either more or less favorable. Either way, as population increases, solutions including zone 3 will eventually catch up and become most optimal. The model's sensitivity to zone 3's population increases abruptly once this point is reached, and can even change sign from a positive to a negative correlation.

In the second test, zone 3 remained inferior to zone 2 even though the two have the same population. And ignoring population, zone 3 is more closely linked to all other zones than zone 2. This discrepancy can be explained by the third factor – zone 3's closest neighbors all have low population, whereas the remaining zones are distant and populous. Thus the model has less sensitivity to population in this situation.

The third test shows that the model is very sensitive to a decrease in zone 1's population. This is an effect constructively caused by all three factors. First, the relative isolation of zone 1 makes it a bad candidate when its population is made average-sized. Second, the reduced population reduces the need for hospitals to be built in zone 1. Lastly, the reduced population weakens its negative impact on the zones that cannot easily reach zone 1 as well as its positive impact on those that can.

The results of the latter two tests show average times similar to the solution to the original problem. As the table above shows, the zones in general have similar geographical proximity scores, on the order of 9 to 11, so population distributions similar to the current distribution should not differ too much in optimal average times. Only the large change in test one made a marked difference.

5.0 Memo to ESC

Dear ESC,

We are high school students who are sincerely concerned about maximizing the number of residents who can be covered within 8 minutes of an emergency call in the country. We understand that there are limited number of ambulances and a great population to be covered. Therefore, we analyzed the given statistics with different models to produce a desirable solution to each situation.

When there are 3 ambulances available, we produced the following results with two different models. First, considering how to maximize the population covered, our first model produced 11 different combinations that can cover everyone within 8 minutes of an emergency call. Since they can all cover the total population, we then continued to consider the combinations with higher efficiency and shorter transit time because they lead to a higher chance of survival for the patients. Through the use of the improved model, out of the 11 solutions, we found out that the combination of zone 1, zone 2 and zone 5, has the highest efficiency. Therefore, we recommend you to put the location of your ambulances in zone 1, zone 2, and zone 5.

For the situation of having only 2 ambulances, we discovered that there is only one possible solution to cover all six population zones within eight minutes of an emergency call. The two different models produced the same result, a combination of zone 2 and zone 5, which confirms that this solution is the only solution and the optimal solution. Therefore, we recommend the bases to be in zone 2 and zone 5 when there are only two ambulances available.

For the situation of having only 1 ambulance, no matter what zone the ambulance is placed, it is impossible to cover all the population. In this case, the only ambulance should be placed at zone 2, since it covers the most population compared to others. The population that can be covered is 160,000, and the population left uncovered is 110,000. Although according to the other model it does not have the highest efficiency, we are more concerned of covering the maximum population, since this is the primary goal of ESC.

As for a catastrophic event, although it is rare, it is necessary to consider how to prepare for such events, because such events have higher potential of chaos and massive damages. We understand your difficulty of preparing for a catastrophic event because one has to cope with the problem when there are many calls from different population zones simultaneously. Thus, for a thorough and precautionary back-up, we have considered such situations and designed two models for you to consider. First, our first model designed a path that crosses all six zones. We concluded that the ambulances should be stationed in zones 1, 3, and 5, and they should all take the common route in emergency. However, with an improved model, we designed 3 small routes that are independent of each other. With these routes, the most optimal locations to station ambulances would be zones 2, 4, and 5. Ambulances there should respond to calls from their own zone as well as from zones 1, 3, and 6 respectively. Considering efficiency (average time required to save a person), we highly recommend you to follow the results of the second model and arrange the ambulances in zones 2, 4, and 5.

At last, we highly recommend you to make sure there are always at least 2 ambulances available in order to cover the whole population. We hope that you can take our recommendations seriously. And we ensure you that our recommended solutions and arrangements will allow you to not only maximize the people covered, but also reach the highest efficiency at the same time.

Thank you

Sincerely,
Team #4185

6.0 Appendix

6.1 Bibliography

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6.2 Java

This program is the brute force program used to confirm the results of the Boolean Method. It gives one optimal solution for 3 ambulances and finds the number of solutions for 1, 2 or 3 ambulances.

```
public class BooleanSum {
    public static final double oo = Double.POSITIVE_INFINITY;
    public static final double[][] TRANSIT_TIMES = {{1,8,12,14,10,16},
                                                    {8,1,6,18,16,16},
                                                    {12,18,1.5,12,6,4},
                                                    {16,14,4,1,16,12},
                                                    {18,16,10,4,2,2},
                                                    {16,18,4,12,2,2},
                                                    {oo,oo,oo,oo,oo,oo}};
    public static final int[] POPULATION =
{50000,80000,30000,55000,35000,20000};

    public static void main(String[] args) {
        int min[] = {0,0,0,numUncovered(0,0,0)}; //Location A, Location B,
Location C, uncovered population
        int solutions3 = 0;
        int solutions2 = 0;
        int solutions1 = 0;
        for (int i = 0; i < 6; i++) {
            for (int j = i+1; j < 6; j++) {
                for (int k = j+1; k < 6; k++) {
                    if (numUncovered(i,j,k) < min[3]) {
                        min[0]=i;
                        min[1]=j;
                        min[2]=k;
                        min[3]=numUncovered(i,j,k);
                    }
                    if (numUncovered(i,j,k) == 0) {
                        solutions3+=1;
                    }
                }
                if (numUncovered(i,j,6) == 0) {
                    solutions2+=1;
                }
            }
            if (numUncovered(i,6,6) == 0) {
```

```
                solutions1+=1;
            }
        }
        System.out.println("          Zone 1     Zone 2     Zone 3     Zone 4     
Zone 5     Zone 6");
        System.out.printf("Uncovered: ");
        for (int i = 0; i < 6; i++) {
            double d = Math.min(TRANSIT_TIMES[min[0]][i], 
Math.min(TRANSIT_TIMES[min[1]][i],TRANSIT_TIMES[min[2]][i]));
            if (d > 8) {
                System.out.printf("%-6d",POPULATION[i]);
            }
            else {
                System.out.print("0     ");
            }
            System.out.printf("(%.0fm) ",d);
        }
        System.out.printf("\n\nChose %d, %d and %d. Total uncovered = %d. 
\nTotal number of solutions with 3 ambulances = %d, with 2 ambulances = %d, with 
1 ambulance = %d", min[0]+1, min[1]+1, min[2]+1, min[3], solutions3, solutions2, 
solutions1);
    }

    public static int numUncovered(int A, int B, int C) {
        int sum = 0;
        for (int i = 0; i < 6; i++) {
            double d = Math.min(TRANSIT_TIMES[A][i], 
Math.min(TRANSIT_TIMES[B][i],TRANSIT_TIMES[C][i]));
            if (d > 8) {
                sum += POPULATION[i];
            }
        }
        return sum;
    }

}
```

This program is used in the weighted average model. It calculates the C table and tests all combinations of 1, 2 or 3 ambulances for the lowest weighted average.

```
public class Checker {
    public static final double[][] TRANSIT_TIMES = {{1,8,12,14,10,16},
                                                    {8,1,6,18,16,16},
                                                    {12,18,1.5,12,6,4},
                                                    {16,14,4,1,16,12},
                                                    {18,16,10,4,2,2},
                                                    {16,18,4,12,2,2}};
    public static final double[] POPULATION =
{50000,80000,30000,55000,35000,20000};
    public static int TotalPopulation = 0;
    public static double[][] CTable = new double[6][6];

    public static void main(String[] args) {
        GenerateContributionsTable();

        //Check the best combination for one
        int minOne = 0;
        for (int i = 0; i < 6; i++) {
            if (wAvgOfOne(i) < wAvgOfOne(minOne)) {
                minOne = i;
            }
        }

        //Check the best combination for two
        int[] minTwo = new int[2];
        for (int i = 0; i < 6; i++) {
            for (int j = 0; j < 6; j++) {
                if (wAvgOfTwo(i,j) < wAvgOfTwo(minTwo[0],minTwo[1])) {
                    minTwo[0]=i;
                    minTwo[1]=j;
                }
            }
        }

        //Check the best combination for three
        int[] minThree = new int[3];
        for (int i = 0; i < 6; i++) {
```

```
            for (int j = 0; j < 6; j++) {
                for (int k = 0; k < 6; k++) {
                    if (wAvgOfThree(i,j,k) <
wAvgOfThree(minThree[0],minThree[1],minThree[2])) {
                        minThree[0]=i;
                        minThree[1]=j;
                        minThree[2]=k;
                    }
                }
            }
        }

        System.out.printf("1 Ambulance:\n"
                        + "The best departure zone is %d; its weighted average
time is %f.\n"
                        + "2 Ambulances:\n"
                        + "The best departure zones are %d and %d; the
combination's weighted average is %f.\n"
                        + "3 Ambulances:\n"
                        + "The best departure zones are %d, %d and %d; the
combination's weighted average is %f.\n",
                        minOne+1,wAvgOfOne(minOne),
                        minTwo[0]+1, minTwo[1]+1,
wAvgOfTwo(minTwo[0],minTwo[1]),
                        minThree[0]+1,minThree[1]+1,minThree[2]+1,
wAvgOfThree(minThree[0],minThree[1],minThree[2]));
    }

    public static void GenerateContributionsTable() {
        for (int i = 0; i < 6; i++) {
            TotalPopulation += POPULATION[i];
        }
        for (int i = 0; i < 6; i++) {
            for (int j = 0; j < 6; j++) {

    CTable[i][j]=TRANSIT_TIMES[i][j]*POPULATION[j]/TotalPopulation;
            }
        }
    }

    public static double wAvgOfThree(int A, int B, int C) {
        double wAvg = 0;
        for (int i = 0; i < 6; i++) {
            wAvg += Math.min(Math.min(CTable[A][i], CTable[B][i]),
```

```
CTable[C][i]);
        }
        return wAvg;
    }

    public static double wAvgOfTwo(int A, int B) {
        double wAvg = 0;
        for (int i = 0; i < 6; i++) {
            wAvg += Math.min(CTable[A][i], CTable[B][i]);
        }
        return wAvg;
    }

    public static double wAvgOfOne(int A) {
        double wAvg = 0;
        for (int i = 0; i < 6; i++) {
            wAvg += CTable[A][i];
        }
        return wAvg;
    }
}
```

This program is used to quickly generate the cycle periods between 2 zones in the split-cycle model.

```
public class OptimalHospital{
    public static final double[][] TRANSIT_TIMES = {{1,8,12,14,10,16},
                                                    {8,1,6,18,16,16},
                                                    {12,18,1.5,12,6,4},
                                                    {16,14,4,1,16,12},
                                                    {18,16,10,4,2,2},
                                                    {16,18,4,12,2,2}};
    public static void main(String[] args) {
        System.out.println("        Zone 1  Zone 2  Zone 3  Zone 4  Zone 5  
Zone 6");
        for (int i = 0; i < 6; i++) {
            System.out.print("Zone "+ (i+1) + "  ");
            for (int j = 0; j < 6; j++) {
                if (i != j) {
                    System.out.printf("%8.0f",emergencyTime(i,j));
                }
                else {
                    System.out.print("       ");
                }
            }
            System.out.println();
        }
    }

    public static double emergencyTime(int a, int b) {
        double intraA = TRANSIT_TIMES[a][a];
        double betweenAB = TRANSIT_TIMES[a][b];
        double betweenBA = TRANSIT_TIMES[b][a];
        return 2*intraA + betweenAB + betweenBA; //circuit length
    }
}
```