Summary

The first quintessential problem we face is to determine the shortest time it takes to commute between different zones, which we solved by employing both the **Dijkstra's Algorithm** and the **Floyd–Warshall Algorithm**, favoring the latter in practice.
Having reasoned that an ambulance arrangement that effectively covers the whole county is one that maximizes the population coverage, we applied the **Tree Traversal** programming method to Questions 1&2, and obtained solutions that succeeded in covering the entire region. We then analyzed the given data and answered Question 3 by putting 1 ambulance in the location with the largest population coverage and calculated the number of people left without coverage.
We optimized the basic model from 2 different angles, building matrixes and making accurate mathematic calculations to achieve both the most efficient way to cover the population and the fastest way to respond to emergency calls. We concluded that if ambulances are arranged in such a way so that the regions they cover overlap, they can provide more secure care in case one of them is dispatched, while the shorter the average time it takes an ambulance to reach designated areas is, the faster its response. Therefore we came up with 2 optimization plans, one to increase coverage overlap, and another to ensure fastest contact. We used standard deviation as a method of determining the best solutions among the optimized ones.
To solve the problem concerning catastrophic scenarios we approached it from 3 perspectives depending on the nature of the catastrophe, and proceeded to discuss 3 different plans on how to station the ambulances so that they most substantially boost rescue rates. In case of sporadic outbreaks in random areas, we applied the aforementioned plan with large area overlaps so that enough ambulances will be on standby once 1 has been dispatched to deal with a crisis case. Our second plan is to have the ambulances repeatedly dispatched and retrieved all over the county, but found that this does not provide satisfactory coverage. Most notably, in cases where 3 ambulances are involved and needs to reach all areas as fast as possible, we used the method of overall planning to determine several ways to station ambulances so that they can travel through all 6 zones within 8 minutes.
We also went into specifics in the catastrophic scenario, and discussed 2 types of possible calamities in detail: the Radiation Model, in which a crisis erupts in a single location and spread throughout the county, downgrading in intensity as it spread, and the Diffusive Model, in which a crisis does not decrease in intensity but rather disseminate. We chose earthquakes to represent our Radiation Model and epidemics to represent our Diffusive Model. Using formulas from **Richter's Magnitude Scale** and the **SIR Model** we compiled 2 constructive plans on how to deal with these disasters.
Having put these specific samples under minute examination, we concluded our models and factor models into a well-organized plan that can be elaborated to determine the best solution for counties and cities, regardless of their land area.

Wee Woo Wee Woo

Ambulance Response Mechanism

Group #4155

Contents

1. Introduction

When some people are enjoying lives with their healthy bodies, others are suffering from illness and pains. In cases of severe injuries and acute diseases, the efficiency of emergency medical response plays a decisive role in saving lives, which is the reason why governments are starting to pay great attention to the establishment of highly-efficient emergency response mechanisms. However, there are still cases where people die from the lack of in-time medical assistance, especially the late arrivals of ambulances. The emergency response mechanism of ambulances is in need of improvement.

Ambulances are vehicles used for the transportation of the sick or injured to, from or between places of treatment and are required in a variety of emergency situations, all of which potentially life threatening, including but not limited to unconsciousness, heart attacks, uncontrollable bleeding, burns, choking, convulsions, strokes, and extreme allergic reactions.

However, according to *the Daily Mail*, although UK possesses one of the fastest emergency medical response mechanisms, only three of UK's 32 ambulance services reach a large majority of “immediately life-threatening” call-outs within 8 minutes, and thousands of people lost their lives because their ambulances took longer than necessary to reach them.

Thus building an effective short-range mechanism to combat crisis should be considered one of the top priorities in the medical field. An optimization of the system would require better preparedness, faster response and more efficient operation.

More importantly, where the ambulances are geographically stationed is of great importance in determining the time it takes for the rescue crew to reach the designated area. A well-situated ambulance station helps to maximize the number of residents that can be reached within a short time of an emergency call.

2. Breaking down the problem

The overarching problem is to find the best locations of ambulances so as to provide emergency medical responses most efficiently.

In this problem we have 2 variables, the population and the area (zones). In order to lessen our variables, we make a logical assumption that if the positioning of the ambulances allows them to cover the whole county, then the population coverage must also at its maximum. This means that in an ideal situation, all the 6 zones, and therefore all residents, are covered with available ambulance(s). Only when available ambulance(s) are unable to cover all zones do demographic differences need to be taken into discussion. We can accordingly simplify our model to contain only one variable: the zones the ambulances cover.

Thus when approaching the problem, we prioritize solutions that cover all zones, and then approach solutions that are based on demographic density.
We make a hypothesis that 3 ambulances, if positioned fittingly, can cover all zones, and proceed to find the possible solutions.

We divided the four problems into two parts:

(1) Question 1, 2 and 3 ask about the best locations of ambulances under a standard situation, which are considered the same in nature and thus can be dealt with in the same approach;

(2) Question 4 asks about the distribution and placement of ambulances when a catastrophic event occurs, which means wider involvement and more urgent need of ambulances.

3. Assumptions

(1) As to the "semi-perfect" condition mentioned in the problem, we assume that it is a condition in which no traffic, weather, or any objective hazards obstruct the ambulances, and in which all ambulances and related personnel are working under perfect condition and the capacity of emergency treatment departments is adequate to meet all the medical response requirement.

(2) We assume that the population of each zone remains relatively stable, i.e. in a state of dynamic balance, and that under no circumstances will massive changes occur.

(3) The "8 minutes" given means eight 60-second minutes, i.e. 480 seconds in all.

(4) Under a catastrophic condition concerning epidemics, which will later be discussed in detail as an example of catastrophic occurrence, we assume that the disease transmission process begins instantly.

4. Variables Clarification

Variable Name	Explanation
i	The zone where the first ambulance is located.
j	The zone where the second ambulance is located.
k	The zone where the third ambulance is located.
$h(x)$	A logic discriminant array consists of six numbers，the initial value being 1.
shuju	A matrix whose rows and columns correspond to the rows and columns of *Table 1* respectively, while i, j, k are all included in its rows.
s	An independent logic discriminant.
P_t	The total population of the county.
P_r	The overlap of the population covered by the ambulance(s).
$P_{(x)}$	The population covered by the x^{th} ambulance.
P_u	The population left without coverage.
$t_{(x)}$	The least time possible it takes to reach *Zone X*.
T_{ijk}	The average time needed to reach each zone when the ambulances are stationed in particular zones.
R_p	The overlap rate of the coverage.
d	The distance from *Zone 1* to another arbitrary zone.
M_L	Local magnitude.
v	Fatality rate.
S	Earthquake intensity.
$E[L]$	The expected number of fatalities.
$x(t)$	A continuous and differentiable function representing the number of affected people at a certain time.
λ	The average effective contact per person.
N	The whole population of certain area.
$s(t)$	The ratio of the susceptible population number to the total population at a certain time.
$i(t)$	The ratio of the infected population number to the total population at a certain time.

5. Model I – Standard Condition

The standard condition refers to a situation in which no wide-scale catastrophe is involved and all emergency responses are carried out according to normal procedures.

5.1 Graphic Analysis

As given in the problem settings, the county is divided into 6 zones, and the average time required to travel between each zone under semi-perfect condition is listed as below:

	Average Travel Times (min.)					
Zones	1	2	3	4	5	6
1	1	8	12	14	10	16
2	8	1	6	18	16	16
3	12	18	1.5	12	6	4
4	16	14	4	1	16	12
5	18	16	10	4	2	2
6	16	18	4	12	2	2

Table 1

The question setting requires that the ambulances arrive within 8 minutes of a 911 call, thus by circling out the average times below 8, we can obtain the zones which each location of ambulance(s) can reach within 8 minutes.

The population of each zone is varied and listed below:

Zones	Population
1	50,000
2	80,000
3	30,000
4	55,000
5	35,000
6	20,000
Total	270,000

Table 2

Through analyzing the data about the population each ambulance can cover when placed in each location, we can build up a conclusion about whether the ambulance(s) can cover all population when positioned in a certain way. Thus establishing the fundamental model on which further analysis can be made.

Going over *Table 1* given in the Problem, we can induce the time required to travel directly from one zone to another. For example, it takes 16 minutes to go directly from *Zone 1* to *Zone 6*. However, unlike what some might suppose, the direct path may not be the shortest one we can take to commute between different zones.

Due to the influence of variables like the layout of the city, highways and bus-way planning, etc., it might be more efficient not to draw a straight line from *Zone 1* to *Zone 6* and take that path, but to take a highway that may wind through *Zone 2* along the way but offer much smoother travel. This is a perfectly probable assumption, and we set out to analyze our data to find out the real shortest time to get from one zone to another.

This idea of more vertexes (i.e. intersections) bringing faster travel is exemplified in the reasoning above. Notably, we did consider the fact that the more vertexes, the more intersections in the pathways, and hence the higher the possibility of traffic accidents. High chances for accidents, however, still does not pose a large threat to the model as a whole, because road accidents in general have slight occurrences and therefore will not undermine the accountability of the model.

Our findings later will correspond with our assumption: the time shown on *Table 1* is not the shortest time it takes to commute between Zones, and we will present here further results of our analysis.

5.2 Main Models

Since the purpose is to maximize the people who can be reached within 8 minutes, the most ideal situation is that all the 6 zones are covered by the available ambulance(s), which means the whole population of the county is within reach of 8-minute-emergency medical response.

In order to make sure whether the average travel times given in *Table 1* are in terms of the most efficient routes, and if not, obtain the real shortest times, we employ Floyd Algorithm Model and Dijkstra Algorithm Model to make further analysis.

5.2.1 Dijkstra Algorithm Model

To calculate the real shortest distance between two Zones, we apply Dijkstra's algorithm. This is an algorithm commonly used in routing processes, and is therefore directly applicable to our problem.

Using this algorithm, we first define a starting point and an ending point, i.e. the two Zones involved, and then trace different pathways expanding outward from the starting point, labeling each intersection we pass along the way. Through a process of screening we repeatedly look for every intersection that is closer to the starting point until we reach the destination. Though the amount of calculation involved makes it a relatively slow process, there is no denying that this algorithm can ultimately help us find the shortest path.

Here are the detailed steps we used to attain the results:

(1) Let $l(u_0) = 0$, while $v \neq u_0$, let $l(u_0) = \infty$, $S_0 = \{u_0\}$, $i=0$.

(2) For each $v \in \bar{S_i}$ ($\bar{S_i} = V - S_i$), replace $l(v)$ with:

$$\min_{u \in S_i}\{l(v), l(u) + w(uv)\}$$

mark this minimum u_{i+1}, and deem that $S_{i+1} = S_i \cup \{u_{i+1}\}$.

(3) Stop when $i=|V|-1$, if $i<|V|-1$, replace i with $i+1$, and turn to (*ii*).

5.2.2 Floyd Algorithm Model

To apply the Floyd Algorithm, we can start with any single unilateral path. The distance between two pinpoints is the weight of the path between them. If 2 points do not have a path connecting them, the weight becomes infinitely large.

Suppose we have a pair of vertices *u* and *v*. If there is a vertex *w* so that the path from *u* to *w* and then to *v* is shorter than the known path we replace our figures.

Next we manifest our thinking with an adjacent matrix, matrix *G*. If we can trace a path between points V_i and V_j, then *G*[*i,j*] has a value, which we will deem as *d*, where *d* represents the distance between *i* and *j*; if we cannot trace a path between V_i and V_j then G[*i,j*] becomes infinitely large.

We use a matrix *D* to record the points we insert into the paths. *D*[*i,j*] represents the points we have to cross when going from V_i to V_j, with the initial value of *D*[*i,j*] being *j*. After we start inserting points into the matrix, we compare the distance after we have inserted a point to the original distance. Let $G[i,j]=\min(G[i,j], G[i,k]+ G[k,j])$. If the value of *G*[*i,j*] decreases, then we deem *D*[*i,j*]=*k*. In Matrix *G* we can find information about the distance between two given points, while in Matrix *D* we have information about the shortest path.

Suppose, for example, that we want to trace a path from V_5 to V_1. Referring to Matrix *D*, if $D_{(5,1)}=3$ then the path from V_5 to V_1 pass through point V_3. The path is $\{V_5, V_3, V_1\}$. If $D_{(5,3)}=3$, then V_5 and V_3 are directly connected. If $D_{(3,1)}=1$, then V_3 and V_1 are directly connected.

The Floyd Algorithm is applicable to all APSP(All Pairs Shortest Paths) Problems. It is a dynamic programming algorithm with best graph effects and weights that can either be positive or negative. This algorithm is simple and effective. Since the triple cycle structure is very compact, the effectiveness of this algorithm greatly surpass that of the Dijkstra's Algorithm, which has to undergo calculation |V| times. It can use simple codes to determine the shortest path between any two arbitrary spots.

Therefore we favored the Floyd Algorithm Model over the Djkstra's method and used it to get our results.

Thus we establish *Table 3* as below, illustrating the relation between Zones and the shortest average travel times.

	Shortest Average Travel Times (min.)					
Zones	1	2	3	4	5	6
1	1	8	12	14	10	12
2	8	1	6	16	12	10
3	12	18	1.5	10	6	4
4	16	14	4	1	10	8
5	18	16	6	4	2	2
6	16	18	4	6	2	2

Table 3

By listing the corresponding population and covered zones of each location, we can combine the above two tables and establish a new table as below, showing the population coverage according to the different locations of ambulances:

Ambulance Location	Covered Zone	Covering Population
1	1,2	130,000
2	1,2,3	160,000
3	3,5,6	85,000
4	3,4,6	105,000
5	3,4,5,6	140,000
6	3,4,5,6	140,000

5.2.3 Traversing Tree Model

To solve the first 2 questions we decide to employ the Traversing Tree model, which is one of the most explicit and inclusive approaches to solving such questions.

The tree traversal refers to "the process of visiting each node in a tree data structure, exactly once, in systematic way", and we apply it to our questions by the following steps:

Suppose we put an ambulance in Zone 1. This ambulance covers Zone 1 & 2, and therefore we need to place ambulances so that they cover Zones other than 1 & 2. The solutions branch out into Zone 2,3,4 and 6. If we put the second ambulance in Zone 2, then we have covered Zones 1, 2 and 3. We need one more ambulance covering Zones 4, 5, 6, so we put the third ambulance in Zone 5. We have now come up with

our first set of solution: putting the ambulances in Zones 1, 2 and 5.
The following diagram showcases part of the thought process:

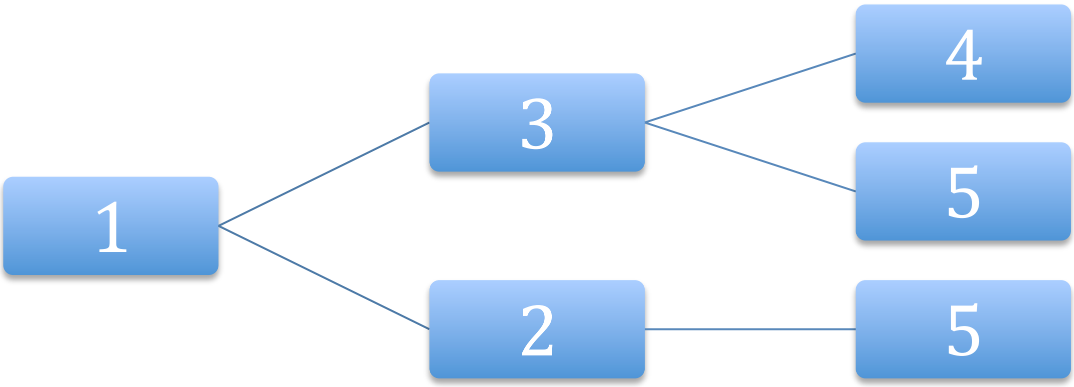

Diagram 1

Based on this search method, we can build a Traversing Tree Model that helps us enumerate all of the possible locations of the 3 ambulances.

(1, 2, 5)	(1, 2, 6)	(1, 3, 4)
(1, 3, 5)	(1, 3, 6)	(1, 4, 5)
(1, 4, 6)	(1, 5, 6)	(2, 3, 4)
(2, 4, 5)	(2, 4, 6)	(2, 3, 5)
(2, 5, 6)	(2, 3, 6)	(1, 5, 5)
(1, 1, 5)	(1, 6, 6)	(1, 1, 6)
(2, 5, 5)	(2, 2, 5)	(2, 6, 6)
(2, 2, 6)		

*The last 8 positioning methods are practicable, but not suggested, since we judge that in a Standard Condition, positioning 2 ambulances in the same zone is not effective. Although these methods can help ambulances reach designated areas within 8 minutes, they are less efficient compared with a wider spread ambulance distribution.

This means that under the last 8 circumstances, only 2 ambulances are needed. Thus the last 8 solutions are better fit as the answers to question two where only 2 ambulances are available.

We hereby list all the possibilities of the ambulance locations where all the 6 zones are within 8-minute-reach of the emergency medical response:
(The represents the zone covered by each ambulance, wile the ※ represents the location of each ambulance.)

Solution 1

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						
No.2		※					270,000
No.3					※		

Solution 2

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No,1	※						
No.2			※				270,000
No.3				※			

Solution 3

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						
No.2			※				270,000
No.3					※		

Solution 4

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						
No.2				※			270,000
No.3					※		

Solution 5

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						270,000
No.2				※			
No.3						※	

Solution 6

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1		※					270,000
No,2			※				
No.3				※			

Solution 7

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1		※					270,000
No.2				※			
No.3					※		

Solution 8

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1		※					270,000
No.2				※			
No.3						※	

Solution 9

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1		※					270,000
No.2			※				
No.3					※		

Solution 10

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1		※					
No.2					※		270,000
No.3						※	

Solution 11

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						
No.2					※		270,000
No.3						※	

Solution 12

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						
No.2			※				270,000
No.3						※	

Solution 13

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1	※						
No.2		※					270,000
No.3						※	

Solution 14

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No.1		※					
No.2			※				270,000
No.3						※	

Solution 15

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No,1	※						270,000
No.2					※		

Solution 16

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No,1	※						270,000
No.2						※	

Solution 17

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No,1		※					270,000
No.2					※		

Solution 18

	Covered Zone & Location						Total Population
Ambulance	1	2	3	4	5	6	
No,1		※					270,000
No.2						※	

As shown in the tables above, there are 18 possibilities altogether.

Thus we come to the conclusion that in the cases of 3 and 2 ambulances available, the ideal locations which all the 6 zones are covered are (1,2,5), (1,2,6), (1,3,4), (1,3,5), (1,3,6), (1,4,5), (1,4,6), (1,5,6), (2,3,4), (2,4,5), (2,4,6), (2,3,5), (2,5,6), (2,3,6) and (1,5), (1,6), (2,5), (2,6) respectively.

5.2.4 Maximum Coverage of Population

Based on the facts given, we can reason that the optimal zone to place an ambulance is one in which it maximizes the population it can reach within 8 minutes. Since we have already come to the conclusion that one ambulance cannot effectively cover all zones in 8 minutes in our previous models, we examine the "location-population" diagram again to find out how many people an ambulance can cover when placed in each respective zone. The zone where the ambulance maximizes its coverage is the one where it is best placed in.

We draw up the following diagram:

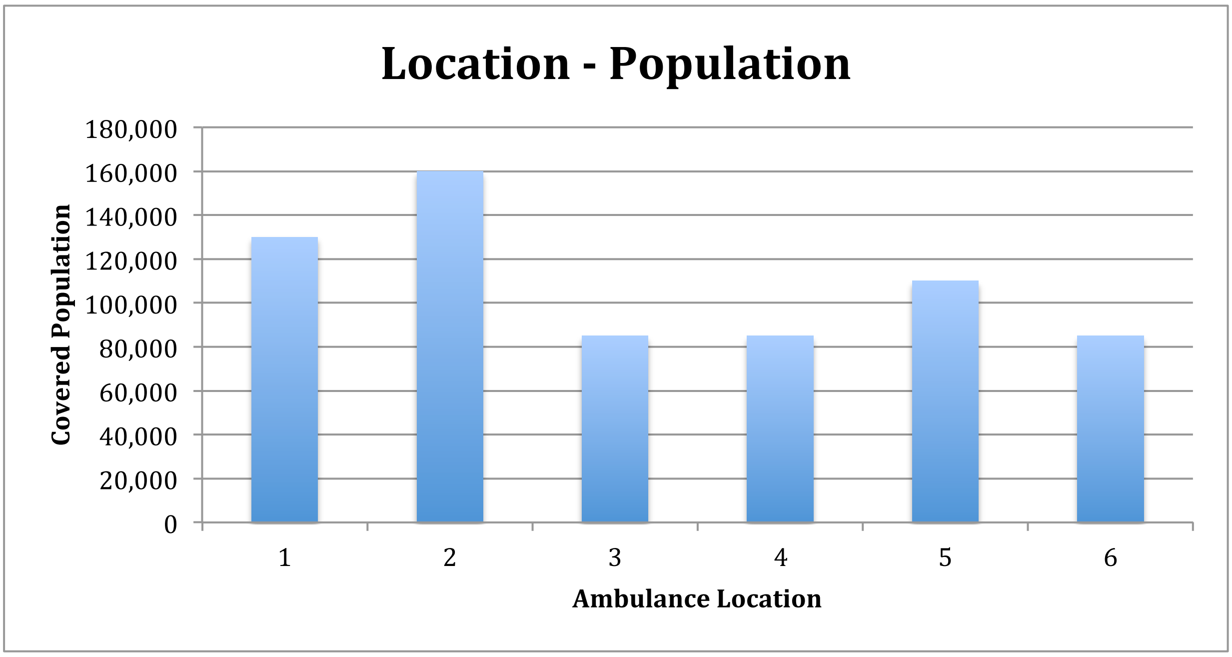

Diagram 2

As is shown in the diagram above, Zone 2 is the optimum choice of location when only one ambulance is available.

However, although Zone 2 is already the best position, we are still unable to cover the whole county. Using the model below, we obtain the population left without coverage:

$$P_u = P_t - P_{(2)}$$

$$= 270{,}000 - 160{,}000$$

Thus we obtain the result that the uncovered population equals 110,000.

5.2.5 Conclusion

(1) When 3 ambulances are available, it is possible to cover everyone in the county within the 8-minute response range, which is also the maximum number of people who can be reached. The locations for the three ambulances have 22 different solutions listed as follows:

(1,2,5), (1,2,6), (1,3,4), (1,3,5), (1,3,6), (1,4,5), (1,4,6), (1,5,6), (2,3,4), (2,4,5), (2,4,6), (2,3,5), (2,5,6), (2,3,6), (1,5,5), (1,1,5), (1,6,6), (1,1,6), (2,5,5), (2,2,5), (2,6,6), (2,2,6)

(2) After one ambulance has been set aside for an emergency call, leaving two ambulances available, we can still ensure that everyone be reached within the 8-minute window, the four locations being:

(1,5), (1,6), (2,5), (2,6)

(3) When two ambulances have both been dispatched, leaving only one ambulance available, the remaining ambulance should be posted at location 2, so as to maximize the number of people within the 8-minute reach. In this case, we can not cover everyone, and the population left without coverage accounts for 110,000

5.3 Optimization

5.3.1 Repeated Coverage

When we look at the data given, we find that both ambulances stationed in Zone 1 and Zone 2 can effectively cover Zone 1 and Zone 2. The same occurs with ambulances stationed in Zone 2 and Zone 3, where both can effectively cover Zone 3, and so on. The overlaps in the model means that an afflicted area can have more than 1 ambulance covering it, and in situations where more than 1 ambulance is needed, an effective back-up can be provided. Thus the higher the overlap rates between population coverage of the ambulances, the more fail-proof their combination.

We used the following equation to determine an optimized plan:

$$\frac{P1+P2+P3-Pt}{Pt} = R_p$$

Where P_1, P_2, and P_3 represent respectively the population covered by the 3 ambulances, P_t represents the total population of the whole county, and R_p represents the overlap rate of the coverage.We thus conclude that the higher the value of R_p, the more efficient the plan is.

The results of this model are listed as below (*Table 4* shows the 3-ambulance occasion while *Table 5* represents the 2-ambulance occasion):

Locations(3 ambulances)	Overlap Rates of the Coverage
(1, 2, 5)	59.3%
(1, 2, 6)	59.3%
(1, 3, 4)	18.5%
(1, 3, 5)	31.5%
(1, 3, 6)	31.5%
(1, 4, 5)	38.9%
(1, 4, 6)	38.9%
(1, 5, 6)	51.9%
(2, 3, 4)	29.6%
(2, 4, 5)	50.0%
(2, 4, 6)	50.0%
(2, 3, 5)	42.6%
(2, 5, 6)	63.0%
(2, 3, 6)	42.6%

Table 4

Locations (2 ambulances)	Overlap Rates of the Coverage
(1, 5)	00.0%
(1, 6)	00.0%
(2, 5)	11.1%
(2, 6)	11.1%

Table 5

By comparing the overlap rates of the coverage, and circling out the locations with the highest overlap rates, we come to the conclusion that the best locations are:

3 ambulances – (2, 5, 6)

2 ambulances – (2, 5) & (2, 6)

5.3.2 Fastest Response

We build a matrix, Matrix *Shuju*, which directly corresponds with *Table 1* given in the Problem. Variables *i, j, k* which head their horizontal lines correspond with the horizontal lines in *Table 1* in representing a zone number. The variable *x*, which heads its perpendicular line also represents a zone number. In effect, the matrix almost exactly resembles *Table 1*.

We first enumerate possible combinations of *i, j, k* with each variable taking on a value from one to six. With each combination of *i, j* and *k*, we also enumerate *x* from 1 to 6. Then we calculate the numerate values of *Shuju(i,x)*, *Shuju(j,x)*, and *Shuju(k,x)* and determine whether any on these are smaller than 8.
If *Shuju(A,B)* is smaller than 8, then we can logically assume that an ambulance located in Zone A can reach Zone B in 8 minutes, which means effective coverage of Zone B is achieved. This means if one of *Shuju(i,x)*, *Shuju(j,x)* and *Shuju(k,x)* has a value smaller than 8, than the area *x* is successfully covered with combination *i, j* and *k*.

Assume that we station 3 ambulances in Zones *i*, *j* and *k*. From the aforementioned Matrix *shuju* we can get the data about how fast each zone can be reached.

We can infer that for *Zone X*, the smallest value amongst *shuju(i,x)*, *shuju(j,x)* and *shuju(k,x)* represents the least time possible it takes to reach Zone X when the ambulances are stationed in *Zone i*, *j* and *k*. We deem this value $t_{(x)}$.

If we add up the data of all 6 zones and divide the sum by 6, we get an average number. Comparing this average time of each *i*, *j* and *k* combination can give us an optimized plan of the basic model: the smaller the sum, the less time it takes to reach the affected population, and therefore the more efficient the plan is.

So we come to the following equation:

$$T_{ijk}=[t_{(1)}+t_{(2)}+t_{(3)}+t_{(4)}+t_{(5)}+t_{(6)}]/6$$

Where T_{ijk} represents the average time needed to reach each zone when the ambulances are stationed in particular zones (i, j, k). By comparing the values of all T and selecting the smallest, we get an optimized plan with time efficiency in consideration.

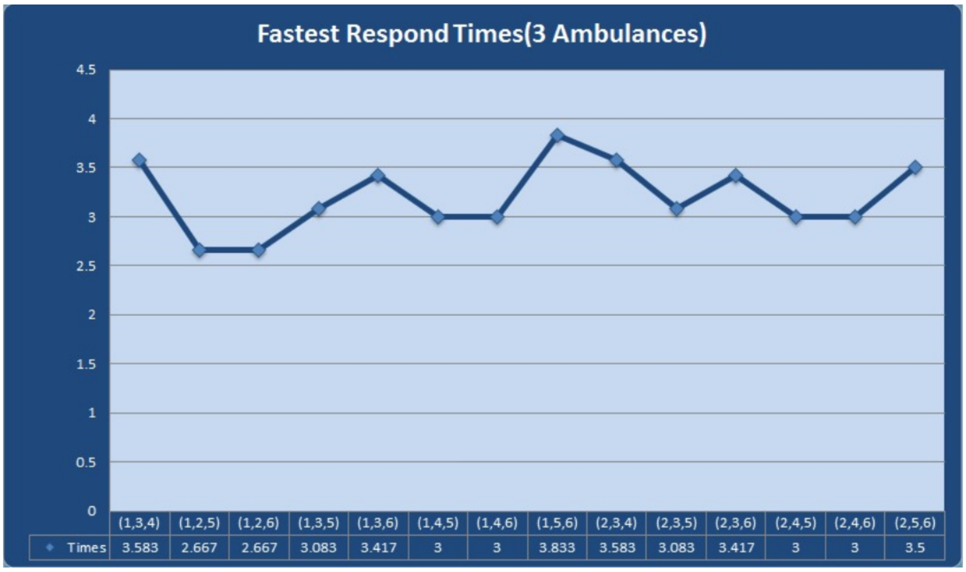

Diagram 3

The above diagram shows the average response time of each location when 3 ambulances are available.

Thus we can easily observe that in terms of response time, the most efficient locations are:

(1, 2, 5) *(1, 2, 6)*

Meanwhile, supposing that the response time is the same, we should bear in mind that the standard deviation of the shortest time it takes to reach each Zone must be small. Since a smaller standard deviation means that the ambulances can reach all Zones relatively quickly, rather than spending a lot of time getting to some locations while reaching other locations in no time.

We use these equations to calculate the standard deviations:

Square Deviation:

$$s^2 = \frac{(x_1 - x)^2 + (x_2 - x)^2 + \cdots (x_n - x)^2}{n-1}$$

Where x represents the average.

Standard Deviation:

$$s = \sqrt{s^2} = \left[\frac{(x_1 - x)^2 + (x_2 - x)^2 + \cdots (x_n - x)^2}{n-1}\right]^{\frac{1}{2}}$$

Since the standard division is the same for the 2 solutions, they are both optimum solutions for the question.

In terms of 2 ambulances, the problem solving procedure is similar. Assume that 2 ambulances are stationed in Zones i and j. Apply it to the *shuju* matrix shown above, we arrive at the model as below:

$$T_{ij} = [t_{(1)} + t_{(2)} + t_{(3)} + t_{(4)} + t_{(5)} + t_{(6)}]/6$$

And then we developed the following *Table 6*:

Location	(1,5)	(1,6)	(2,5)	(2,6)
Average Time	3.833	3.833	3.833	3.833

Table 6

From the above table, we can see that when 2 ambulances are available, all the four locations have the same average response time.

Thus all four are optimum solutions:

(1, 5) *(1, 6)* *(2, 5)* *(2, 6)*

As with plans involving 3 ambulances, we also calculated the standard deviation of these 4 solutions and, finding them to be the same, decides that all four are optimum solutions.

6. Model II – Catastrophic Condition

Catastrophic condition stands as an extreme out of the Standard Condition. When a catastrophe occurs, which mainly includes natural disasters such as earthquakes, tsunami, and infectious diseases, usually more population and wider areas are involved, which means a much more intensive need for emergency medical response of ambulances than under the Standard Condition is expected.

In this case, the placement and dispatch mechanism of ambulances needs readjustment and more factors should be taken into consideration.

6.1 General Model

In a catastrophic situation in which a massive population has been affected, the symptoms of distress may break out all at once in all parts of the county and among all the population, or they might spring up from a central spot and proceed to spread throughout the area, or perhaps they just start cropping up in random places sporadically.

6.1.1 Overlap Coverage

As we have mentioned earlier on, in several ambulance arrangements the population coverage of the ambulances overlap with one another. This overlap becomes extremely useful in a catastrophic situation where victims appear sporadically and in random places.

Since no general outbreak is expected, the ambulances will be alert on standby. Once one of the three responds to a call and goes into action, the other two will hopefully still maintain effective coverage of the whole county, so that they can also give immediate response once another case erupts in a different place.

In this case, the solutions we give above for the optimized model of repeated coverage is also the optimum solution for this type of catastrophic situation.

6.1.2 Come-and-Go

Another way to cover the situation is by the repeated dispatching and returning, i.e. coming back and forth, of the three ambulances so as to accommodate the need of as many people as possible.

Referring to *Table 3*, by adding up the average traveling time accordingly, we establish the following table, showing the average time needed traveling back and forth for once between two zones:

Zones/Average Traveling time (min)	1	2	3	4	5	6
1	2	16	24	30	28	28
2		2	24	30	28	28
3			3	14	12	8
4				2	14	14
5					4	4
6						4

Table 7-1

Since we are required to respond within 8 minutes of all emergency calls, any solution with an average traveling time of over 8 minutes is not effective. We replace those solutions with the sign "∞" to give a more explicit illustration.

Zones/Average Traveling time (min)	1	2	3	4	5	6
1	2	∞	∞	∞	∞	∞
2		2	∞	∞	∞	∞
3			3	∞	∞	8
4				2	∞	∞
5					4	4
6						4

Table 7-2

By analyzing *Table 7-2*, we listed all the possible come-and –go routes and the other zones that can be covered after the ambulance return from each route accordingly.

Possible Route (Zone⇌Zone)	Supplementary Coverage (Zone)
1⇌1	N/A
2⇌2	3
3⇌3	3
4⇌4	6
5⇌5	3
6⇌6	4 or 6
3⇌6	3 or 5
5⇌6	(when originally located in 5) 4
	(when originally located in 6) 3

Table 7-3

Using the aforementioned traversing tree model, we search for the solutions that can cover all 6 zones within the effective response time (in our case the given 8 minutes). We conclude that it is impossible to cover all zones in the come-and –go situation with 3 ambulances.

6.1.3 Three Ambulances Coverage

In case of extreme catastrophic situation where massive destruction erupts all over the county, we need to get all 3 ambulances on spot as soon as possible. More than that, we had better make sure that an ambulance, when stationed in an appointed area and travelling along an appointed path, can go through all 6 Zones within 8 minutes to constructively boost rescue chances.

	Average Travel Times (min.)					
Zones	**1**	**2**	**3**	**4**	**5**	**6**
1	1	8	∞	∞	∞	∞
2	8	1	6	∞	∞	∞
3	∞	∞	1.5	∞	6	4
4	∞	∞	4	1	∞	8
5	∞	∞	6	4	2	2
6	∞	∞	4	6	2	2

Table 8-1

Table 8-1 showcases the average time to commute between Zones. For the sake of clear illustration we have replaced all time more than 8 minutes with the infinity sign ∞, and these are expelled from consideration because of their evident ineffectiveness.

Location	Route Plan	Time	Total Time
1	1→2	8	8
2	2→1	8	8
	2→3	6	6
3	3→5→6	6+2	8
	3→6→5	4+2	6
4	4→6	8	8
	4→3→6	4+4	8
5	5→4→3	4+4	8
	5→3	6	6
	5→6→4	2+6	8
	5→6→3	2+4	6
6	6→3	4	4
	6→4	6	6
	6→5→3	2+6	8
	6→5→4	2+4	6

Table 8-2

In Table 8-2 we list all routes between Zones that can be completed in 8 minutes. These chunks of route plans allow us to get a clearer picture about how to effectively plan an overall course for the ambulances, so that they can traverse through all Zones within 8 minutes.

Ambulance	Route Plan	Average Time
1, 2, 5	1→2, 2→3, 5→6→4	7.33
1, 2, 6	1→2, 2→3, 6→5→4	6.66
1, 3, 4	1→2, 3→5→6, 6→4	7.33
	1→2, 3→6→5, 6→4	6.66
	1→2, 3→5→6, 6→5→4	7.33
	1→2, 3→6→5, 6→5→4	6.66
1, 3, 5	1→2, 3→5→6, 5→4→3	8.00
	1→2, 3→6→5, 5→4→3	7.33
	1→2, 3→5→6, 5→6→4	8.00
	1→2, 3→6→5, 5→6→4	7.33
1, 3, 6	1→2, 3→5→6, 6→4	7.33
	1→2, 3→6→5, 6→4	6.66
	1→2, 3→5→6, 6→5→4	7.33
	1→2, 3→6→5, 6→5→4	6.66
1, 4, 5	1→2, 4→6, 5→4→3	8.00
	1→2, 4→6, 5→3	7.33
	1→2, 4→6, 5→6→3	7.33
	1→2, 4→3→6, 5→6→4	8.00
	1→2, 4→3→6, 5→6→3	7.33
	1→2, 4→3→6, 5→3	7.33
	1→2, 4→3→6, 5→4→3	8.00
1, 4, 6	1→2, 4→6, 6→5→3	8.00
	1→2, 4→3→6, 6→5→3	8.00
	1→2, 4→3→6, 6→5→4	7.33
2, 3, 4	2→1, 3→5→6, 4→6	8.00
	2→1, 3→5→6, 4→3→6	8.00
	2→1, 3→6→5, 4→6	7.33
	2→1, 3→6→5, 4→3→6	7.33
2, 3, 5	2→1, 3→5→6, 5→4→3	8.00
	2→1, 3→5→6, 5→6→4	8.00
	2→1, 3→6→5, 5→4→3	7.33
	2→1, 3→6→5, 5→6→4	7.33
2, 3, 6	2→1, 3→5→6, 6→4	7.33
	2→1, 3→5→6, 6→5→4	7.33
	2→1, 3→6→5, 6→4	6.66
	2→1, 3→6→5, 6→5→4	6.66

2, 4, 5	2→1, 4→6, 5→4→3	8.00
	2→1, 4→6, 5→6→3	7.33
	2→1, 4→6, 5→3	7.33
	2→1, 4→3→6, 5→4→3	8.00
	2→1, 4→3→6, 5→3	7.33
	2→1, 4→3→6, 5→6→4	8.00
	2→1, 4→3→6, 5→6→3	7.33
2, 4, 6	2→1, 4→6, 6→5→3	8.00
	2→1, 4→3→6, 6→5→3	8.00
	2→1, 4→3→6, 6→5→4	7.33
2, 5, 6	2→1, 5→4→3, 6→3	6.66
	2→1, 5→4→3, 6→4	7.33
	2→1, 5→4→3, 6→5→3	8.00
	2→1, 5→4→3, 6→5→4	7.33
	2→1, 5→3, 6→4	6.66
	2→1, 5→3, 6→5→4	6.66
	2→1, 5→6→4, 6→3	6.00
	2→1, 5→6→4, 6→5→3	8.00
	2→1, 5→6→3, 6→4	6.66
	2→1, 5→6→3, 6→5→4	6.66

Table 8-3

This final graph, Table 8-3, is our final response to the problem. On the left side we list the probable solutions, where the ambulances can be stationed, to be exact, while in the middle we draw out the respective paths that these ambulances can trace in an emergency so that they can course through all 6 zones within 8 minutes. On the right side we give the corresponding time to each path plan. The optimum plan for any catastrophic occurrence can therefore be determined using this graph despite their geographical differences.

6.2 Intensity Distribution

As for the most extreme catastrophic condition, where the three ambulances are all obliged to be dispatched at the same time, we need to place and dispatch the ambulances according to the influence intensity so as to build up the most effective and efficient ambulance response system.

Since spreading patterns and intensity difference vary according to the types of the disasters, we break down the discussion into two respective conditions.

6.2.1 Radiation Model

The radiation model applies to disasters that spread its impact in a radiation way, which means the center of the occurrence has the most casualty rates and therefore needs the most ambulances, while the intensity of other areas decrease progressively.

The most typical case in the radiation model is earthquakes, thus we hereby take earthquakes as an example to establish the model.

Disregarding factors like road planning, we assume that the time it takes to travel between two Zones is directly proportional to the distance between them.

Suppose an earthquake happens in *Zone 1*, and the distance from Zone 1 to another arbitrary Zone (for the sake of the hypothesis we will use Zone 2 as an example) is d. The distance from *Zone 1* to the other 4 Zones becomes accordingly K_1d, K_2d, K_3d and K_4d.

In comes the Richter Magnitude Scale.

The Richter magnitude of an earthquake is determined from the logarithm of the amplitude of shock waves recorded by seismographs.

The original formula is:

$$M_L = log_{10} A - log_{10} A_0(\delta) = log_{10} [A/A_0(\delta)], \quad (1)$$

Where A is the maximum excursion of the Wood-Anderson seismograph, the empirical function A_0 depends only on the epicentral distance of the station δ, and M_L stands for the Local Magnitude.

Using this equation we come to the following results:

$$M_1 = log\ A$$

$$M_2 = log\ A - log\ A_0\ (d)$$

While for the other 4 Zones we have:

The constant A, which needs inputting, is the magnitude of the earthquake at the epicenter. Since the $\log A_0$ (d) in all M_k equations are equal in value, the earthquake magnitude in different Zones can be thought of as a ratio between 2 constants minus different known numbers.

The second equation we consult deals with the earthquakes' shaking intensity and their corresponding fatality rates. This is the Empirical Fatality Rate Equation developed by Jaiswal et al (2009), a new global empirical model where the fatality rate (v), which is the function of shaking intensity (S), can be expressed in terms of a two-parameter lognormal distribution function as follows:

$$v(S) = \Phi[\frac{1}{\beta}\ln(\frac{S}{\theta})] \quad (2)$$

Where Φ is the standard normal cumulative distribution function. The fatality rate dep ends on the two free parameters of the cumulative distribution function of the logno rmal distribution namely, θ and β.

Because most parameters involved are constants, the ratio of fatality rate between different zones can be conversed into ratios of their respective ($\ln S$-$\ln\theta$), where S represents the intensity of the earthquake.

Suppose all factors beyond the shaking intensity remains constant all over the county, then the ratio between the shaking intensity is the ratio between the magnitudes.

This means that if the earthquake magnitude in *Zone A* is k times the magnitude in *Zone B*, the fatality rate in *Zone A* is larger than that of Zone B by $\ln k$.

After we figure out the ratio of fatality rates, i.e. the ratio of v, we can use *Model 3* to determine the fatality rates in each Zone.

$$E_i[L] \approx \sum_j v_i(S_j)\ P_i\ (S_j) \quad (3)$$

Where $P_i(S_j)$ denotes an estimated population exposed to shaking intensity S_j for an event i. Then the expected number of fatalities $E[L]$ can be denoted as the above.

From all the points above we can come to the conclusion that the nearer a place is to the epicenter, the higher the casualty rate, which is to say that the nearer a place is to the center of the earthquake, the higher its demand for ambulances. The optimum solution in this case is one that can get the ambulances to our affected areas as fast as possible.

Remember the Fastest Response section we did on our optimization plans? Now we can go back and use that model again.

Returning to Matrix *shuju*, we can infer that for *Zone X*, the smallest value amongst *shuju(i,x)*, *shuju(j,x)* and *shuju(k,x)* represents the least time possible it takes to reach Zone X when the ambulances are stationed in *Zone i, j* and *k*. We deem this value *t(x)*.

For each combination of *i, j* and *k*, we calculate their respective *t(1), t(2) , t(3), t(4), t(5)* and *t(6)*. The combination that can get to X in the shortest time is the solution that we are looking for.

Thus we arrive at the solutions as follows:

Occurrence Location	Ambulance Location
1	1,2,6(1,2,3)
2	2,3,6(2,3,6)
3	1,3,5;1,3,6(3,6,5)
4	1,3,4(4,3,6)
5	5,4,1(5,6,3)
6	6,4,1(6,5,4)

Table 9

In *Table 8*, solutions without “()” are the optimum solutions which can arrive the fastest at the Zones in the most need of help, while ensuring a whole-county coverage; those solutions with “()” are the solutions which only achieve the fastest reach to the major disaster area, however without covering the whole county.

6.3.2 Epidemic Model

Define $x(t)$ as a continuous and differentiable function, representing the number of affected people at a certain time t. Assume that the disease transmission begins immediately.

Use λ to represent the average effective contact per person (the number of people infected through contacts).

To examine the increase of patient number between t to t+Δt, we establish the following model:

$$x(t+\Delta t)-x(t)=\lambda x(t)\Delta t$$

$$\Longrightarrow \frac{x(t+\Delta t)-x(t)}{\Delta t}=\lambda x(t) \quad (1)$$

If t=0, let the number of patients be x_0. When Δt →0, we come to the following differential equation:

$$\begin{cases} \frac{dx}{dt} = \lambda x \\ x(0) = x_0 \end{cases}$$

$$\Longrightarrow x(t)=x_0 e^{\lambda t} \quad (2)$$

According to the above model, $x(t)$ keeps increasing to an infinite degree, which is not possible in reality. Notice that only healthy people can be taken into the susceptible pool, and the number of healthy people is in a process of gradual declining. Thus we modify the above model.

Divide the whole population into two groups, namely the susceptible and the infected. Let the number of the whole population be N, and the ratio of the susceptible population number to N and the infected population number to N at certain time be $s(t)$ and $i(t)$.

The number of healthy people infected by each infected person should thus be: $\lambda s(t)$; and the total number of newly infected healthy people each day would be: $\lambda N s(t) i(t)$. Thus we come to the modified model as follow:

$$\begin{cases} N\dfrac{di}{dt} = \lambda N s(t) i(t) \\ s(t) + i(t) = 1 \\ i(0) = i_0 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{di}{dt} = \lambda i(t)[1 - i(t)] \\ i(0) = i_0 \end{cases}$$

$$\Rightarrow i(t) = \frac{1}{1+(\frac{1}{i_0}-1)e^{-\lambda t}} \quad (3)$$

When $i = 0.5$, $\dfrac{di}{dt}$ reaches its maximum, and

$$t_m = \lambda^{-1} \ln(\frac{1}{i_0} - 1) \quad (4)$$

This is when the increase of infected people is at its peak. Local hospitals and the government should be paying special attention at this point!

λ indicates the sanitary condition of the county. Since t_m is inversely proportional to λ, the lower λ is, the better the sanitary condition is. Improving the sanitary condition can serve to postpone the peak of infection.

Notice that in the above *Model 4*, when $t \to \infty$, $i \to 1$, which means the whole county is to be infected. This is very unlikely to happen in reality. In order to make the model more practical, we take the recovered population into consideration, and come to the following adaptation:

Assume the recovered people accounts for μ of the total infected population, where μ is a constant. The recovered people is eligible to second-time infection afterwards.

Thus the average transmission period of this type of disease is $\frac{1}{\mu}$.

The modified model is as follows:

$$\begin{cases} N\dfrac{di}{dt} = \lambda N s(t) i(t) - \mu N i(t) \\ s(t) + i(t) = 1 \\ i(0) = i_0 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{di}{dt} = \lambda i(t)[1 - i(t)] - \mu i(t) \\ i(0) = i_0 \end{cases}$$

$$\Rightarrow \ i(t) = \begin{cases} [\frac{\lambda}{\lambda-\mu} + \left(\frac{1}{i_0} - \frac{\lambda}{\lambda-\mu}\right) e^{-(\lambda-\mu)}]^{-1} & \lambda \neq \mu \\ (\lambda t + \frac{1}{i_0})^{-1} & \lambda = \mu \end{cases} \quad (5)$$

Let $\sigma = \lambda/\mu$, thus σ represents the average number of effective contacts of each infected person within one infection period.

Model 5 can thus be modified as follows:

$$\frac{di}{dt} = -\lambda i(t)[i(t) - \left(1 - \frac{1}{\sigma}\right)] \quad (6)$$

Contact number σ is a domain value. When $\sigma > 1$, the increase and decrease of *i(t)* depends on the value of i_0, but its limit $i(\infty)=1-\frac{1}{\sigma}$ increases if σ increases; if $\sigma \leq 1$ then *i(t)*, the ratio of the infected, decreases and gets closer to zero. This explains why during an epidemic the number of people infected can never exceed the number of original patients.

Most epidemic diseases like smallpox, influenza, hepatitis and measles make the patient immune to them once he or she has recovered from it. The recovered patients are therefore neither healthy nor infected. They are, in effect, totally separated from the whole system. This brings more complications, as we will discuss in the following paragraphs.

Assume that while the disease is spreading, the total population in the given area remains stable. Neither births and deaths nor immigration and emigration are taken into account. We categorize the population into those who are healthy, those who are infected, and those who have successfully recovered from and thus will remain immune to the disease. Deem their respective portion in the total population as *s(t)*, *i(t)*, and *r(t)*; daily contact rate of the infected will be constant λ, daily recovery rate μ, and $\sigma = \lambda/\mu$ the contact number during the epidemic.

From the assumption we induce that $s(t)+i(t)+r(t)=1$. For the number those who have successfully recovered we can use the following equation:

$$N \frac{dr}{dt} = \mu N i(t) \quad (7)$$

Deem the initial ratio of the healthy population and the infected population as s_0 and i_0 respectively, and the initial number of recovered patients $r_0=0$. We thus can get the following differential equation:

$$\begin{cases} \frac{di}{dt} = \lambda s(t)i(t) - \mu i(t), i(0) = i_0 \\ \frac{ds}{dt} = -\lambda s(t)i(t), s(0) = s_0 \end{cases} \quad (8)$$

The above equation is what we will center our model on. Because we cannot determine the analytic solutions of *s(t)* and *i(t)*, we can only use numerical evaluations, although that can be remedied in practice by employing mathematic software, or we can analyze the relationship between *s* and *t* in an *s*-*t* plane.

In this model, $\sigma = \lambda/\mu$ should be recognized as a very important parameter. Since the equation has no analytic solutions, the values of both λ and μ are hard to estimate. However after an epidemic outbreak we can gather the value of s_0 and s_∞ to calculate σ with this model:

$$\sigma = \frac{\ln s_0 - \ln s_\infty}{s_0 - s_\infty} \quad (9)$$

When similar epidemics hit again, presuming that λ and μ does not go through too much change, we can use this value of σ to determine the process of the epidemic transmission.

With the above models, we establish the following diagram.

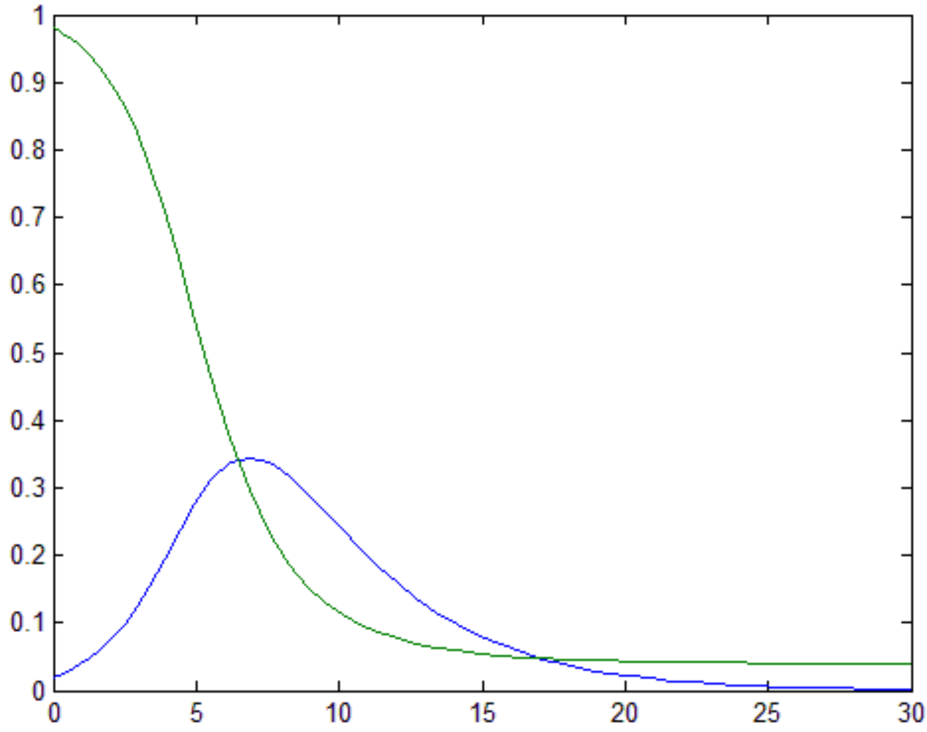

Diagram 4

Using the models above, we conclude that the optimum solution under this circumstances is

(1, 2, 5)

7. Analysis and Application

7.1 Model Analysis

7.1.1 Strengths Analysis

Using our basic models and all factor models, we can generate several solutions about where to station ambulances in different scenarios. Since our models cover both standard and catastrophic occurrences, and expand on several specific cases in the catastrophic categories, it is reasonable to conclude that they are both comprehensive and adequately effective. All our answers are products of careful analysis and induction: some of the answers, in fact, are produced by using different models on the same questions, therefore they are trustworthy and fault-proof.

Not only are they reliable, the models also present room for further elaboration, in that they can also be applied to counties and cities in a broader sense, and are not limited only to the questions dealt with in the problems. Several models have been optimized a number of times, and the calculation processes involved are as simple and direct as we can make them.

7.1.2 Improvement Analysis

However, as with all models, we did not forget that important assumptions have been made during the building of those models, especially with the standard models concerning Question 1,2&3, where the condition is "semi-perfect". Sifting carefully through the whole emergency response mechanism, we can see that there are still many variables and uncertainties left out of consideration, and, in fact, cannot possibly be added into calculation since they are so varied and erratic.

We still have our minds on, for example, the call taking time, the delays that might occur in dispatching processes, the time period between the dispatching of the crew and the start of the vehicle, the health and capability of the crew members, whether extreme road and weather conditions are present, or whether government policies change in times to crisis...and the list goes on. These, however, we recognize to be the icing on the cake. Due to time limits, these variables had to be left out in the modeling process, though we plan to expand on them in the future.

7.2 Application

We build a final model that can be utilized in any scenario where 3 ambulances are on standby. Once we insert our figures about the time it takes to commute between all zones into a matrix, an instant optimum result of where to station the 3 ambulances can be obtained. In this optimized version of *Model 1*, the variable of *h(x)* can also be self-defined, which means that the Zone number does not necessarily needs to be 6. The model therefore, is not solely applicable to this county, but can also be used with any other region, with however many Zones involved depending on the actual situation.

We deem *h(x)* as a logic discriminant array consisting of six numbers，with its the initial value being 1.

If *Zone X* can be covered by the combination of *i,j* and *k*, then we multiply *h(x)* by 1, so *h(x)* remains 1. If *Zone X* cannot be covered by this combination then we multiply *h(x)* by 0, so *h(x)* becomes 0.

We deem *s* as :

$$s=h(1)*h(2)*h(3)*h(4)*h(5)*h(6)*\ldots\ldots*h(x)$$

If all values from *h(1)* to *h(x)* are 1, then *s*=1, in which case we conclude that the combination of locating ambulances in *Zones i, j,* and *k* can reach all *x* zones. If one of the values is 0, then *s*=0, which means that the combination cannot reach all *x* zones. This combination is therefore not an effective solution and subsequently then discarded.

8. Recommendation Memo

Dear Sir/Madam:

Looking over the data concerning ambulance distribution and emergency control in your county, we realize that a more effective short-range mechanism to combat regional crisis is needed. We believe that better situated ambulance stations can help maximize the number of residents that can be reached after emergency calls, and having built several mathematic models that helped us gain fair conceptions as to how to improve arrangements, we put the following information and advice at your disposal:

Under normal conditions, we propose the following two ways of arranging your three ambulances so as to maximize the population they can reach in 8 minutes:

Plan One

If you prefer the most fail-proof arrangement, which means you can cover all the population in your county at all times to the highest degree, then plan 1 is what you would be looking for. The detailed station plan is as follows:

1. When all the 3 ambulances are available, station them at (2, 5, 6). Thus you can cover all the zones of your county, and the overlap coverage rate (OCR) in terms of the population is 63%, which means 63% of the people in your county have more than 1 ambulance that can reach them within 8 minutes.
2. When 1 ambulance has already been dispatched, and there are 2 ambulances available, place them at (2, 5) or (2, 6). All zones are covered with both location arrangements offer the OCR of 11%. (This also means that the first ambulance to be dispatched should be the one stations at 5 or 6.)
3. When 2 ambulances have been dispatched, and only 1 ambulance is left available, unfortunately, it is impossible to cover all the zones. However, the best location for the last ambulance is 2, with 160,000 people within the 8-minute reach.

Plan Two

If you regard the response speed or efficiency as priority, then you might want to consider Plan Two. The detailed station plan is as follows:

1. When all the 3 ambulances are available, station them at (1, 2, 5) or (1, 2, 6), since these two arrangements have the shortest average respond time among all the arrangements that are able to cover the whole county.
2. When 1 ambulance has already been dispatched, and there are 2 ambulances available, place them at (1, 5), (1, 6), (2, 5) or (2, 6). They all have the shortest average respond time among all the arrangements that are able to cover the whole county.
3. When 2 ambulances have been dispatched, and only 1 ambulance is left available, the situation is the same with Plan One, the best location is 2, and 160,000 people

are within the 8-minute reach.

In other rare cases, such as when a catastrophe occurs, we have developed another set of models to design the optimum arrangement of the three ambulances. However, since the situation varies according to different types of diseases and different major disaster locations, we are unable to list all of the possible arrangements here. You are welcomed to contact us at any time for more information if you are interested.

The model we used to gather all these data is a scientifically sound and logically refined one, built out of detailed analysis and careful observation.

Through our proposal, we hope to increase the outreach of the ambulances, strengthen their coordination, and accelerate the process from receiving emergency calls to giving fast and appropriate responses. If these can be achieved then the capacity of rescue missions can be increased, and rescue approaches will be more tailored to the specific needs of the local population.

We also suggest you consider other measures to boost your emergency response mechanism. We strongly recommend you invest more capital in the whole system and upgrade your ambulance arrangement by keeping more standby vehicles on hand so as to ensure effective coverage of the whole county.

For more extensive optimizations, we also suggest engrafting more strategically planned capital-intensive infrastructure such as smart grids and traffic networks on a regional basis. Perhaps large-scale emergency drills can be put into practice so that local communities are involved early in planning and all details are taken into consideration at the designing stage.

It is our sincere hope that our models can be of good reference. Thank you for your attention.

Best regards.

Sincerely,
HIMCM Team #4155

9. Appendix

9.1 Reference

(1) PAUL W. RENO, Factors Involved in the Dissemination of Disease in Fish Populations, Journal of Aquatic Animal Health 10:160–171, 1998

(2) K.S. Jaiswal1 D.J. Wald2 P.S. Earle2 K.A. Porter3 and M. Hearne, Earthquake Casualty Models Within the USGS Prompt Assessment of Global Earthquakes for Response (PAGER) System, University of Cambridge UK June.15-16, 2009

(3) http://en.wikipedia.org/wiki/Dijkstra's_Algorithm

(4) http://en.wikipedia.org/wiki/Richter's_Magnitude_Scale

(5) http://en.wikipedia.org/wiki/Tree_traversal

(6) JOSE´ BADAL,w, MIGUEL VA´ ZQUEZ-PRADA and A ´LVARO GONZA´ LEZ, Physics of the Earth, University of Zaragoza, Sciences B, Pedro Cerbuna 12, 50009 Zaragoza,Spain,Preliminary Quantitative Assessment of Earthquake Casualties and Damages

(7) http://en.wikipedia.org/wiki/Floyd-Warshall_algorithm

9.2 Matlab Code

（1）

```
#dijkstra#
clear;
M=10000;
a(1,:)=[2,16,18,4,12,2];
a(2,:)=[zeros(1,1),1,8,12,14,10];
a(3,:)=[zeros(1,2),1,6,18,16];
a(4,:)=[zeros(1,3),1.5,12,6];
a(5,:)=[zeros(1,4),1,6];
a(6,:)=[zeros(1,5),2];
a=a+a';
pb(1:length(a))=0;pb(1)=1;
index1=1;
```

```
index2=ones(1,length(a)); d(1:length(a))=M;d(1)=0;
temp=1;
while sum(pb)<length(a)
    tb=find(pb==0);=
    d(tb)=min(d(tb),d(temp)+a(temp,tb));
    tmpb=find(d(tb)==min(d(tb)));
    temp=tb(tmpb(1));
    pb(temp)=1;
    index1=[index1,temp];
    index=index1(find(d(index1)==d(temp)-a(temp,index1)));
    if length(index)>=2
        index=index(1);
    end
    index2(temp)=index;
end
d, index1, index2
```

（2）

```
h=zeros(1,6);
data=[1,8,12,14,10,12;8,1,6,16,12,10;12,18,1.5,10,6,4;16,14,4,1,10,8;18,16,6,4,2,2;16
,18,4,6,2,2];
for i=1:1:6
     for j=1:1:6
          for k=1:1:6
             for x=1:1:6
                if data(i,x)<=8 || data(j,x)<=8 || data(k,x)<=8
                    h(x)=h(x)*1;
                else h(x)=h(x)*0;
                end
             end
             s=h(1)*h(2)*h(3)*h(4)*h(5)*h(6);
             if s==1 && i~=j && j~=k && i~=k
                     i;
                     j;
                     k;
                     s=0;
                     su=su+1;
             end
             h=ones(1,x);
            end
      end
end
su
```

（3）

```
#fast response#
i=1;
sum=0;
min=zeros(1,6);
data=[1,8,12,14,10,12;8,1,6,16,12,10;12,18,1.5,10,6,4;16,14,4,1,10,8;18,16,6,4,2,2;16
,18,4,6,2,2];
a=input('ambulance1=');
b=input('ambulance2=');
c=input('ambulance3=');
for i=1:1:6
        if data(a,i)<=data(b,i)
                min(i)=data(a,i);
        else min(i)=data(b,i);
                if min(i)>data(c,i)
                        min(i)=data(c,i);
                end
        end
end
min
for i=1:1:6
        sum=min(i)+sum;
end
aver=sum/6
```

（4）

```
#ill#
function y=ill(t,x)
a=1;
b=0.3;
y=[a*x(1)*x(2)-b*x(1);-a*x(1)*x(2)];

[t,x]=ode45('ill',ts,x0);
plot(t,x(:,1),t,x(:,2)), grid,pause
plot(x(:,2),x(:,1)),grid on;
end
```

（5）

```
# ambulance situation#
function ambulance
for i=1:1:x
        for j=1:1:x
                for k=1:1:x
                    for x=1:1:x
```

```
            if shuju(i,x)<=8 || shuju(j,x)<=8 || shuju(k,x)<=8
                h(x)=h(x)*1;
            else h(x)=h(x)*0;
            end
          end
          s=h(1)*h(2)*h(3)*h(4)*h(5)*h(6);
          if s==1 && i~=j && j~=k && i~=k
                i;
                j;
                k;
                s=0;
                su=su+1;
          end
          h=ones(1,x);
        end
    end
end
su
```

（6）

```
#8 minutes#
clear;
M=10000;
a(1,:)=[1,8,M,M,M,M];
a(2,:)=[8,1,6,M,M,M];
a(3,:)=[M,M,1.5,M,6,4];
a(4,:)=[M,M,4,1,M,8];
a(5,:)=[M,M,6,4,2,2];
a(6,:)=[M,M,4,6,2,2];
a=a+a';
pb(1:length(a))=0;pb(1)=1;
index1=1;
index2=ones(1,length(a));
d(1:length(a))=M;d(1)=0;
temp=1;
while sum(pb)<length(a)
   tb=find(pb==0);
   d(tb)=min(d(tb),d(temp)+a(temp,tb));
   tmpb=find(d(tb)==min(d(tb)));
   temp=tb(tmpb(1));
   pb(temp)=1;
   index1=[index1,temp];
   index=index1(find(d(index1)==d(temp)-a(temp,index1)));
```

```
    if length(index)>=2
        index=index(1);
    end
    index2(temp)=index;
end
d, index1, index2
```

（7）

```
#Floyd#
D=A;
n=length(D);
for i=1:n
    for j=1:n
        R(i,j)=i;
    end
end
for k=1:n
     for i=1:n
        for j=1:n
            if D(i,k)+D(k,j)<D(i,j)
                D(i,j)=D(i,k)+D(k,j);
                R(i,j)=R(k,j);
            End
         End
     End
     hl=0;
     for i=1:n
        if D(i,i)<0
            hl=1;
           break;
         end
     end
    if(hl==1)
        fprintf('minus')
        break;
end
end
```