Letter to Commissioner of the Department of Wildlife

November 11, 2012

Dear Commissioner of the Department of Wildlife,

As environmentalists ourselves, we enjoyed the opportunity to investigate the efficacy of elk reintroduction in the Great Smoky Mountains National Park and determine how to maximize the population growth of elk.

We are pleased to report that the Manitoban elk population in the Great Smoky Mountains National Park will indeed survive. Our mathematical model is broken down into five smaller variables: birth rate, predation, sickness, accidents/unknown deaths, and poaching. The equation for each variable was calculated separately with statistical analysis based on both the data provided and outside research. When combined, all the variables come together in one overarching model for population growth. Over the past ten years, elk population has increased from the original 25 to today's 140. We predict that in the next five hundred years, elk population will drastically increase. Since we calculated the current carrying capacity of the national park to be 1900 elk, numbers will level out at around 1900 elk.

In order to improve the survival of the elk population, we then analyzed the effects of altering each variable and devised a method to increase the carrying capacity of the Great Smoky Mountains National Park. We recommend that three preventative measures be taken: to reduce sickness, reintroduce elk at intervals, and reduce the number of accidents that occur. First, areas where the risk of exposure to parasitic disease is high, especially those located solely in the East that the Manitoban elk are vulnerable to, should be fenced off. Second, to reduce inbreeding and increase the birth rate, new elk from the West should be introduced periodically. Third, we recommend increasing visitor awareness by including a cautionary paragraph in brochures and posting this information in visitor centers.

Again, we appreciate this opportunity and hope that you are able to follow our recommendations in order to increase the number of elk in the Great Smoky Mountains National Park. If so, we fully expect a thriving, permanent elk population in the next 500 years.

Sincerely,

Zi-Ning Choo, Mary Do, Grace Li, and Summer Wu
Illinois Math and Science Academy

Executive Summary

In the winter of 2001, twenty-five Manitoban Elk, native to Canada and the Western United States, were reintroduced into the Eastern United States. However, because the elk were foreign inhabitants in this new region, there are many factors which would affect their survival in the new environment. Our task was to analyze the success of reintroducing the elk to the Great Smoky Mountains National Park and determine how to optimize the results. If the model predicts that the elk population will survive, we would have to consider how to improve the growth of the elk population over time, and if the model predicts that the elk population will die out, we would have to devise a plan to prevent it

Our method consisted of a two-part solution: first, creating a model to predict the future of the elk population and determine whether or not the elk will survive in their new habitat, and second, proposing ways to improve the growth of the elk population based on that model. Using the data we were given, we formulated an equation to relate the birth rate and total death rate of a given year to the population size the previous year. The total death rate function was determined by adding up the equations for each of the different causes of death - predation, accidents, poaching, sickness, and unknown causes. The function for population growth was found by subtracting the total death equation from the birth equation. Population growth is the derivative of population with respect to time, so we were left with an unsolvable differential equation. Consequently, we used Euler's method and a step size of 1 year to derive a graph for population over time. We used a step size of 1 year because elk can only give birth once a year and the data was given in yearly increments.

From our population growth model, we determined that the elk would adjust well to their new surroundings and survive in their habitat. It projects that their population will increase until leveling off at a carrying capacity of about 1900 elk. From our model for death rate, we determined that the two causes that contributed the most to the number of deaths - accidents/unknown and sickness - could be limited by 1) lowering the speed limit, 2) increasing driver awareness, 3) fencing off areas highly populated with disease-causing agents, and 4) administering the appropriate drugs as needed both to prevent and to treat diseases. In addition, to limit the harmful effects of inbreeding, we also recommend introducing five Manitoban elk from the West to the Great Smoky Mountains National Park population every ten years.

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1. Introduction

In the 1800s, Eastern Elk (*Cervus canadensis canadiensis*), which once roamed the North American continent, went extinct as a result of industrialization. Ever since President Theodore Roosevelt signed the bill declaring Yellowstone National Park to be the first national park, efforts have been made to preserve existing fauna and reintroduce species to their natural habitats. In 2001, the Great Smoky Mountains National Park (GSMNP) introduced 25 Manitoban Elk, which belong to the same family as Eastern Elk, to its park in order to a) explore the feasibility of reestablishing an indigenous species, and b) to establish the presence of a large herbivore in order to promote natural ecological processes.[1]

However, because the Manitoban Elk is native to the Western United States, its adaptations in disease tolerances, foods, and environmental preferences differ from those of the original Eastern Elk. In this paper, we build a model to evaluate the efficacy of the GSMNP's reintroduction program and propose a plan to maximize the number of elk over the course of 500 years.

2. Assumptions and Justifications

Assumption: The number of elk that die from unknown causes, sickness, accidents, and predation increases as the elk population increases.
Justification: Even if the elk start developing immunity, pathogens will likewise evolve. Moreover, disease transmission increases with population density. Therefore, the number of deaths from sickness should increase as population increases. Additionally, the probability that an accident or unknown death will occur would naturally increase if there are more elk in the same area. Finally, increases in predator population follow increases in prey population, so more elk will die from predation as their numbers grow.

Assumption: The data points from the years that new elk are reintroduced can be ignored.
Justification: These elk were not part of the local habitat for the entire year; the very first reintroduction came in February.[15] Furthermore, we do not know what time of year the reintroductions in the following years occurred, and whether or not any of the elk were pregnant before their reintroduction. Thus, we felt that including data from the years in which elk were reintroduced would skew the data.

Assumption: The data points at which no elk died as a result of a specific cause were ignored
Justification: The purpose of this model is to determine whether the elk population will stabilize or die out over time. In analyzing the specific causes of death and ignoring the data points at which no elk died, we are over counting the number of deaths. If the model predicts that the elk will survive over the course of 500 years, despite our overcompensation for deaths, our model will have greater credibility.

Assumption: Accidents and unknown deaths can be combined into one cause.
Justification: Accidents and unknown deaths are both random, unpredictable occurrences. Since they both lack a root cause, they can be combined.

Assumption: An elk will die when it reaches age 13.
Justification: According to the National Park Service on the Smoky Mountains website[16], Elk can live as long as 15 years, and typically live an average of 10 to 15 years in the wild. We averaged 10 and 15, to get 13, and assumed that was the average lifespan of an elk.

Assumption: Once all the possible causes of death are factored in, no elk will die from old age.
Justification: As animals age, they physically weaken, and are more susceptible to factors such as sickness and predation. In the wild, animals typically die from these factors before they can die from old age.

Assumption: There will be no major environmental disasters in the future.
Justification: We have no way of predicting such disasters, which can wipe out entire thriving populations.

Assumption: Trends in the data with which we were provided are representative of future trends.
Justification: We cannot otherwise create a model to predict future elk population size.

3. Variables and Parameters

P = Elk Population
t = Time in years
B(P) = The number of births in a given year as a function of elk population the previous year
S(P) = The number elk who die of sickness in a given year as a function of elk population the previous year
AU(P) = The number elk who die from unknown or accidental causes in a given year as a function of the elk population the previous year
R(P) = The number of elk who die from predation in a given year as a function of the elk population the previous year
C = The number of elk that are illegally poached each year
D(P) = The total number of elk that die in a given year as a function of the elk population the previous year

4. Part I: Population Growth Over Time

4.1 Births

We determined B(P) by finding the Least-Squares Line describing the relationship between births, the response variable, and population size, the explanatory variable, obtaining the following equation, with $R^2 = 0.83$, implying a moderately strong linear fit:

$$B(P) = 0.192 * P + 0.15$$

Our scatterplot and residuals plot (shown in the image below) suggest that a linear model appears appropriate for the data provided, because there are no trends in the residuals (Fig. 1).

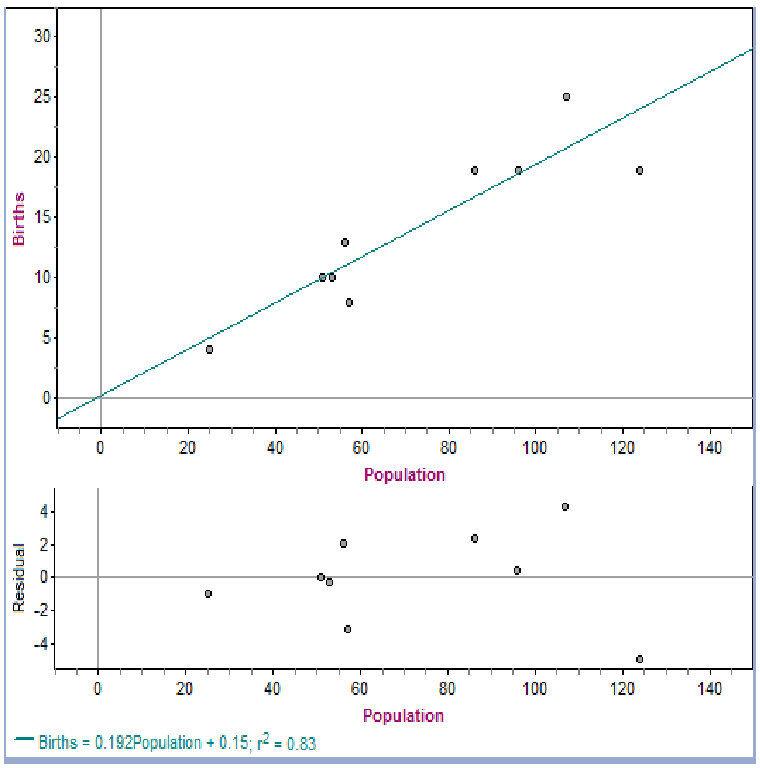

Figure 1. Linear Regression and Residuals Plot for the Projected Birth of Elk.

However, a linear model is in fact not appropriate for describing births with respect to population size, because a population with such a birth rate would grow exponentially with no carrying capacity. Additionally, as elk are a K-selected species, their birth rate likely declines as population density increases. Thus, we decided to reexpress the data by taking the square root of population size and again finding the Least-Squares Line, which had $R^2 = 0.84$.

$$B(P) = 3.21 * \sqrt{P} - 12.6$$

The residuals again showed no trends, suggesting that this model was appropriate for the data provided (Fig. 2).

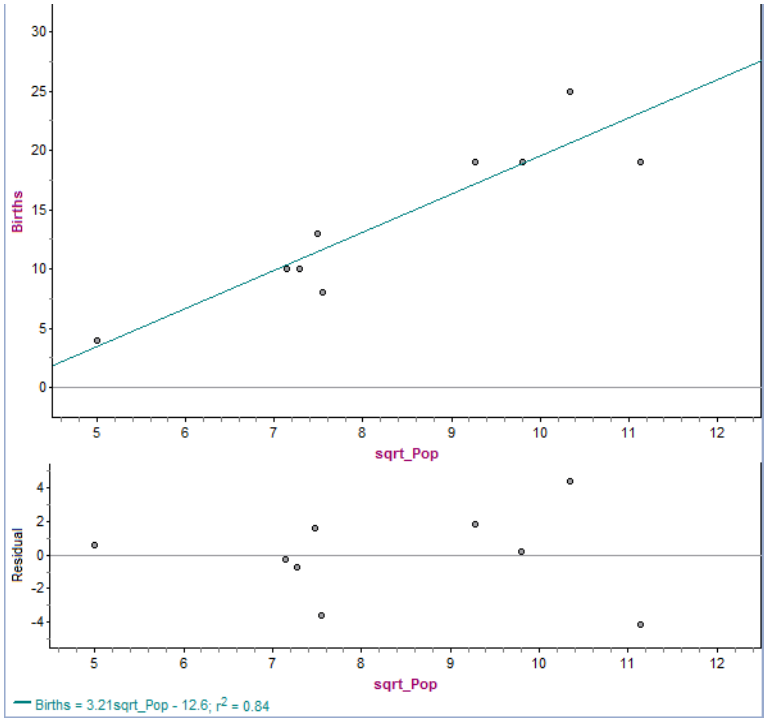

Figure 2. The Linear Regression and Residuals Plot for Births vs. Square Root of Population.

4.11 Preliminary Model: Considering Births and Deaths as a Result of Old Age

Before factoring in the various causes of death, we created a model that only considered the number of births and the number of deaths from old age. Previously, the assumption was made that the lifespan of an elk is 13 years. We started the model in the year 2011, because by using the birth data provided, we would have more accurate numbers for the number of elk that would die from old age for the first decade of our model. A recursive program (see appendix) was written using the following equation, where P_k is the population k years after 2011:

$$P_k = P_{k-1} + B(P_{k-1}) - B(P_{k-13})$$

When the program was run for 175 years, the population was seen to grow, and then level out, as shown here:

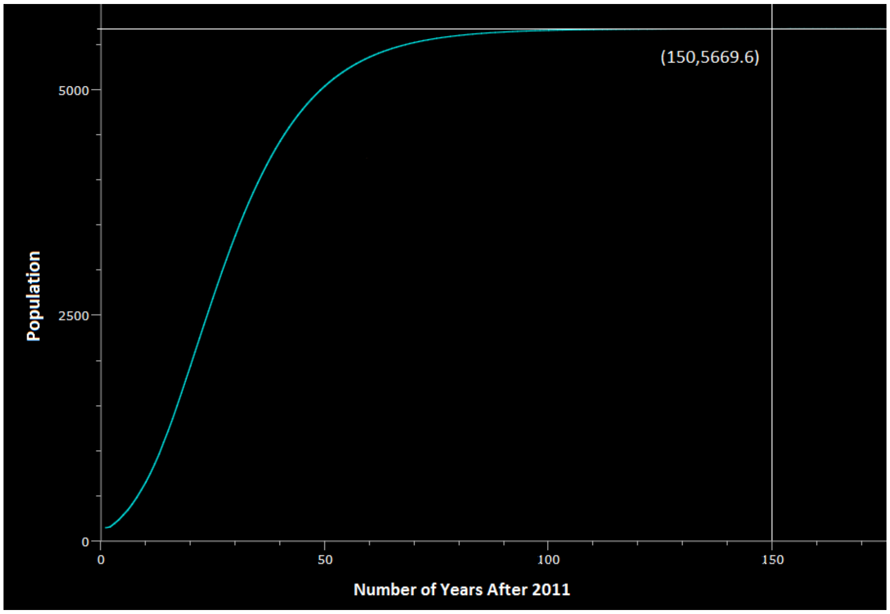

Figure 3. Model of Population vs Time with Just Births and Old Age as Factors.

This preliminary model demonstrates that the elk population will survive and stabilize after reaching a carrying capacity of about 5670 elk. Because this model does not account for any causes of death besides old age, it sets an upper limit on elk population growth. Once other causes of death are incorporated, the elk population must either level off, or die out, and stabilize to a size not exceeding 5670 elk.

4.2 Deaths By Poaching

We then created a model accounting for all the causes of elk death. First, we found that in the given data, the highest number of deaths caused by poaching each year was one. Poaching shows an association with neither the population size nor any of the other variables, and it can be concluded that the number of poachers should stay the same regardless of population size. Thus, we decided that poaching should be accounted for in our model as a constant, which we determined by averaging the number of poaching-related deaths (3) over the number of years for which we were provided data (11), obtaining the following value:

$$C = 0.2727$$

4.3 Deaths By Sickness

Next, we analyzed the deaths resulting from sickness using linear regression and generated the scatterplot shown below with R^2=0.69, which indicates a moderately strong relationship.

$$S(P) = 0.0198 * P + 0.789$$

When creating the scatterplot for this relationship (Fig. 4), we removed one data point: the number of deaths from sickness in 2004. The exceptionally high number of deaths in this year was likely caused by the stress of the elk's introduction to a new environment and their lack of immunity to new diseases. Retaining this point would result in a Least-Squares Line that is not representative of future trends in sickness. Because there are no patterns in the residuals plot, the linear model appears to be a good fit for the data.

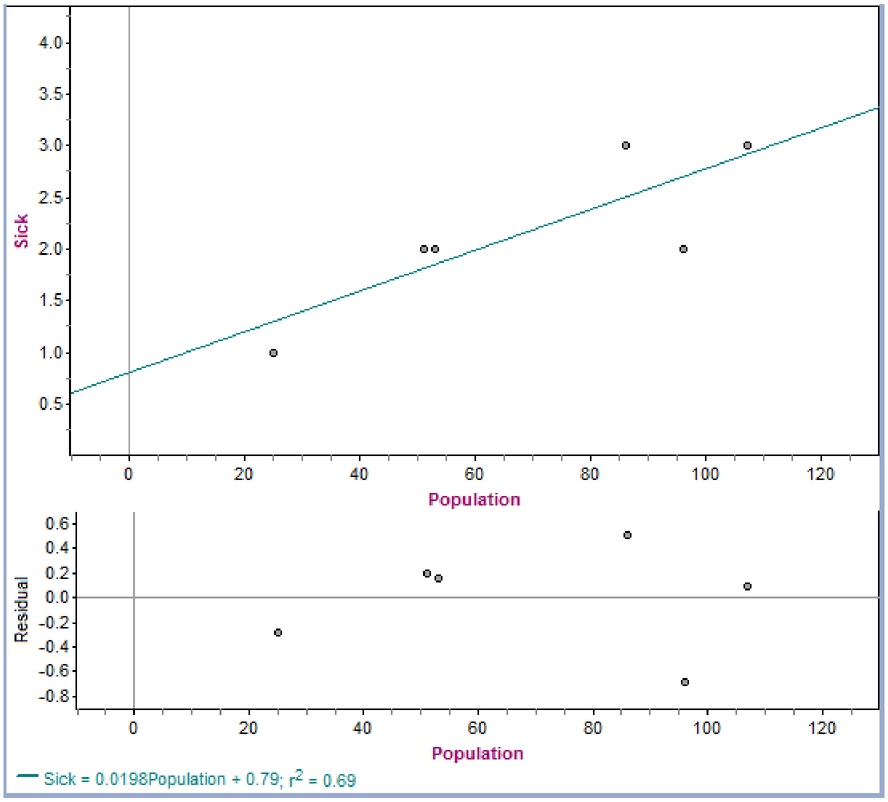

Figure 4. Linear Regression and Residuals Plot for Projected Deaths Caused by Sickness.

4.4 Unknown Causes of Death and Accidents

We then modeled deaths resulting from accidents and unknown causes as a function of population size, generating the equation shown below, with R^2=0.26. Although this value indicates a weak correlation, the randomness in the residuals plot suggests that this linear fit was appropriate (Fig. 5).

$$AU(P) = 0.0311 * P + 0.4399$$

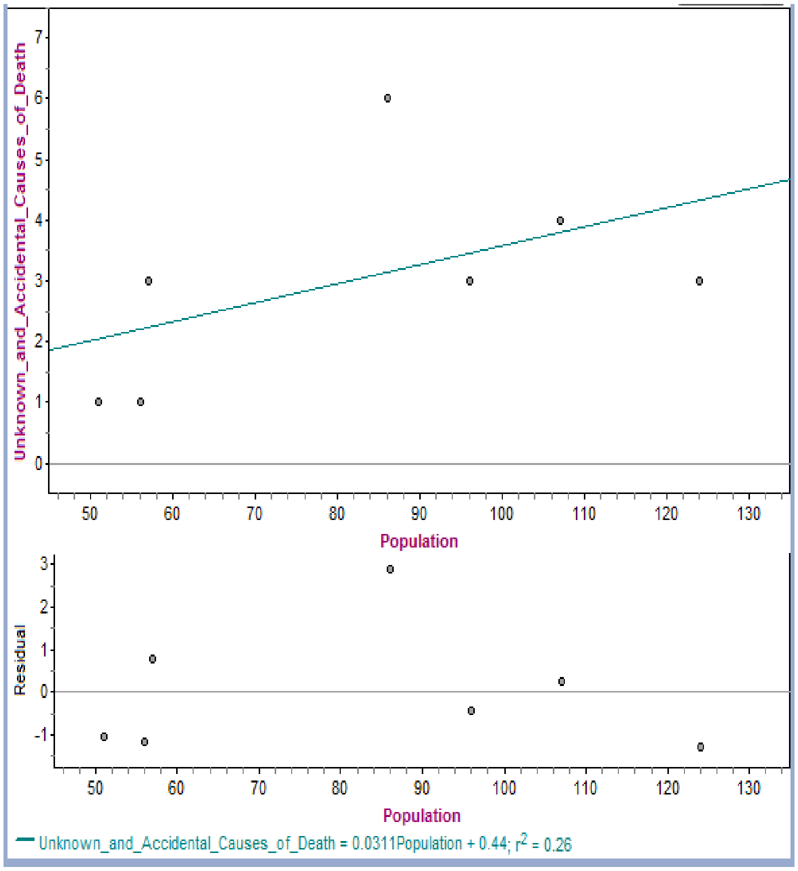

Figure 5. Linear Regression and Residuals Plot for Projected Deaths from Accidents and Unknown Causes.

4.5 Deaths By Predation

Finally, we found an equation relating the deaths resulting from predation to population size, which had R^2=0.49, implying a moderately strong correlation between the two variables. When performing a linear regression, we eliminated the data point from 2005, during which five deaths from predation occurred. According to the National Park Service, newly introduced elk tend to be more susceptible to predation because they have not yet developed predator-avoidance behaviors.[5] We believe that the exceptionally high number of deaths from predators in 2005 likely resulted from the fact that the elk were newly introduced and had not yet adapted to black bear predation in the Smoky Mountains; thus, retaining the data point would result in a regression line that does not correspond to overall trends. Our final scatterplot showing deaths by predation with respect to population size is displayed below. The randomness of the residuals suggests that the linear fit is appropriate (Fig. 6).

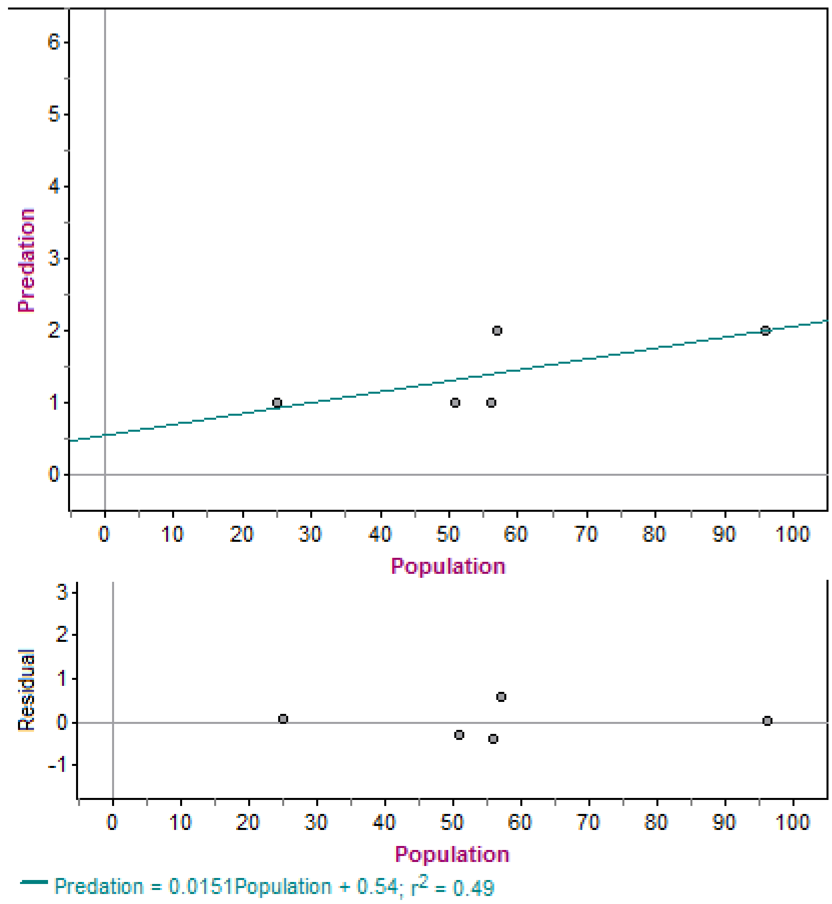

Figure 6. Linear Regression and Residuals Plot for Projected Deaths by Predation.

4.6 Combined Model

We then modeled the change in population size in a given year as a function of the population size of the previous year by combining the equations for births and causes of death that were previously obtained. First, we compiled the separate equations for each distinct cause of death to obtain an all-encompassing equation for elk death:

$$D(P) = S(P) + R(P) + AU(P) + C$$
$$D(P) = (0.0198 * P + 0.789) + (0.0151 * P + 0.539) + (0.0311 * P + 0.4399) + 0.2727$$
$$D(P) = 0.066 * P + 2.0417$$

To make a model for population growth (dP/dt), the birth and total death models were combined. Births cause the population to increase and deaths cause the population to decrease, so the overall population growth is the number of deaths subtracted from the number of births.

$$Population\ Growth = B(P) - D(P)$$

$$\frac{dP}{dt} = 3.21\sqrt{P} - 12.6 - (0.066 * P + 2.0417)$$

$$\frac{dP}{dt} = 3.21\sqrt{P} - 0.066 * P - 14.6417$$

4.7 Vensim 500-Year Projection

After obtaining equations relating births and causes of death to population size, we created a model on Vensim (downloaded from www.vensim.com) to simulate how population size would change over a 500 year period. The diagram below shows the relationships between each of the auxiliary variables and population size (Fig. 7).

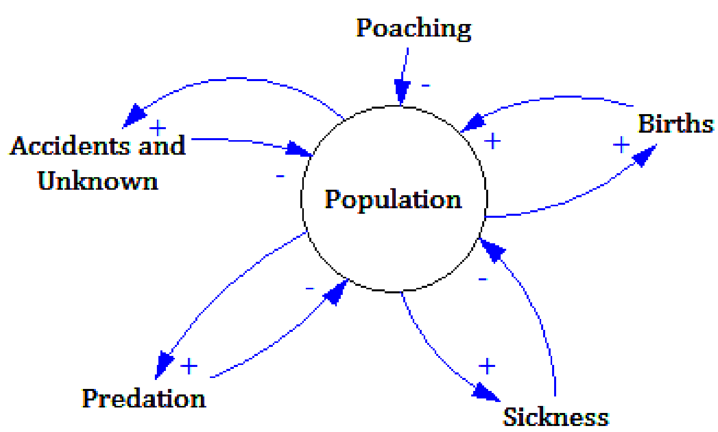

Figure 7. Vensim Model for Population Change.

The model essentially predicted population size using the differential equation from 4.6, which was obtained by subtracting the equations for each of the causes of death from the equation for birth. This differential equation predicts that the environment's current carrying capacity is approximately 1900 elk, a value determined by solving for P at dP/dt = 0.

$$\frac{dP}{dt} = 3.21\sqrt{P} - .066P - 14.6414273$$

We used Euler's method with a step size of one year, because the data given to us was tabulated yearly. We used an initial population value of 86 elk, the size of the elk population after the last introduction of elk. After this point, the size of the elk population was subject entirely to the factors described in our model. The population size change over a 500-year period is depicted in the graph below (Fig. 8).

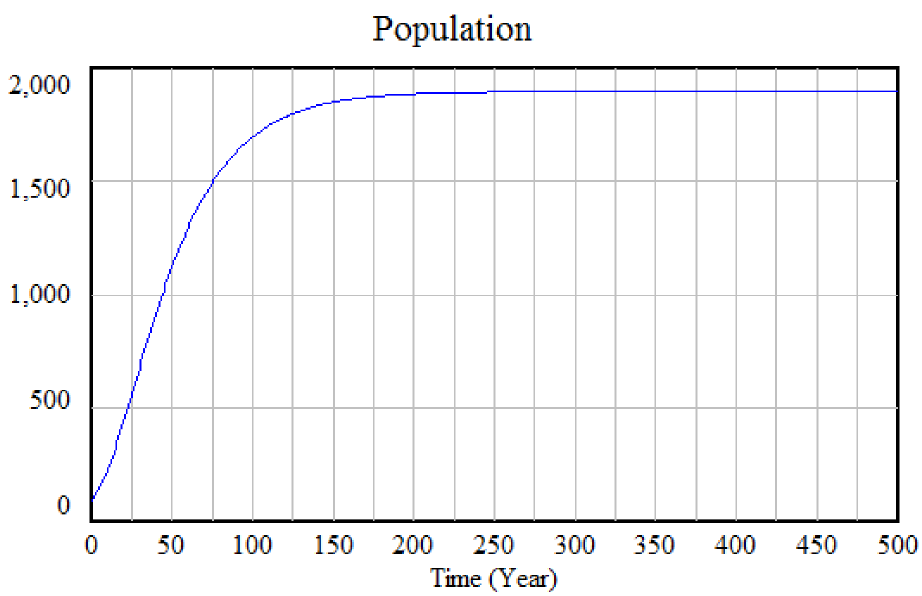

Figure 8. Projected Population Change using Euler's Method on Vensim.

5. Sensitivity Analysis

We conducted a sensitivity analysis to determine how fluctuations in each of the five parameters would affect carrying capacity, which is the maximum population size under a set of given conditions. We modified our equations for Poaching, Sickness, Accidents, and Predation such that rates for each of the causes of death increases by 5%, and the birth rate decreases by 5%, as shown in the following table:

Parameter	Original Equation for Parameter	% Change to Parameter	Modified Equation
Poaching (C)	$C = 0.2727$	+5%	$C = 0.2864$
Sickness (S(P))	$S = 0.0198P + 0.7898$	+5%	$S = 0.02079P + 0.8293$
Births (B(P))	$B = 3.21\sqrt{P} - 12.6$	-5%	$B = 3.0495\sqrt{P} - 14.0114$
Accident (AU(P))	$AU = 0.0311P + 0.4399$	+5%	$AU = 0.0327P + 0.4619$
Predation (R(P))	$R = 0.0151P + 0.539$	+5%	$R = 0.0159P + 0.5660$

Figure 9. Changes in Parameters and Their Resulting Equations.

We then found modified differential equations for each parameter change and used these new DE's to determine the percent change in carrying capacity (K_m). These data are shown in the table below.

Parameter	Modified DE	New Carrying Capacity (K_m)	% Change in K_m
Poaching (C)	$\frac{dP}{dt} = 3.21\sqrt{P} - 0.066P - 14.65506$	1895.4	-0.032
Sickness (S(P))	$\frac{dP}{dt} = 3.21\sqrt{P} - 0.0670P - 14.6809$	1831.6	-3.398
Births (B(P))	$\frac{dP}{dt} = 3.0495\sqrt{P} - 0.066P - 14.0114$	1682.8	-11.245
Accident (AU(P))	$\frac{dP}{dt} = 3.21\sqrt{P} - 0.0676P - 14.6634$	1797.5	-5.194
Predation (R(P))	$\frac{dP}{dt} = 3.21\sqrt{P} - 0.0668P - 14.6687$	1846.7	-2.601
All causes of death increase by 5%	$\frac{dP}{dt} = 3.21\sqrt{P} - 0.0693P - 13.4615$	1735.3	-8.492
All parameters change	$\frac{dP}{dt} = 3.0495\sqrt{P} - 0.0693P - 14.091$	1502.2	-20.780

The results of our sensitivity analysis show that our model is very sensitive to fluctuations in birth rate. A 5% decrease in birth rate produces an 11.2% decrease in K_m; likewise, a 5% increase in all death rates produces only an 8.5% decrease in K_m , but once a 5% decrease in birth rate was further introduced, K_m dropped 20.8%, suggesting that birth rate in the deer population must be monitored carefully. With regard to fluctuations in poaching, predation, and sickness, our model is fairly robust, producing a change in carrying capacity below 5% when a 5% increase in any of those parameters was introduced.

6. Part 2: Increasing Carrying Capacity

6.1 Reducing Sickness

According to the Environmental Assessment for the Experimental Release of Elk in the Great Smoky Mountains National Park[1], because Manitoban Elk are non-native to the Smoky Mountains, they are most susceptible to three main diseases.

1. Parelaphostrongylus tenuis

According to the Kentucky Department of Wildlife, *Parelaphostrongylus tenuis*, a parasitic disease which occurs in Eastern regions, killed around 1% of the reintroduced elk population[1]. To reduce this number, we propose fencing off areas of the Smoky Mountains National Park with the highest concentrations of snails and slugs, as they have the potential to infect elk with the parasites. We also propose that avermectins should be administered to the elk during seasons of high snail and slug activity. Avermectins kill snail and slug larvae before they can reach the elk's central nervous system, but should not be given to the elk regularly as the parasite may develop a resistance to the drug[8].

2. Elaeophora schneideri

These parasitic worms impair blood flow to the head and neck, resulting in blindness, neurological problems, and necrosis of the ears and muzzle. Since the main transmitters of these parasites are horseflies, there are no effective preventative measures. Therefore, we suggest treatment as the course of action for limiting this disease. Since administering diethylcarbamazine would kill off the worms but run the risk of clogging the arteries of the elk with the dead worms, we recommend giving piperazine salts instead, which have been found to be more effective[9,10]. However, no treatment is known for the cerebral form of the disease.

3. Babesia odocoilei

Distributed throughout southeastern United States, this parasite causes a malaria-like disease and is spread by ticks. We recommend administering imidocarb dipropionate, which is a drug that has been known to consistently clear the parasite from the blood.[11]

6.2 Preventing Inbreeding

Elk are polygynous; one male will mate with a harem of up to twenty females. Because polygynous mating systems are more vulnerable to the harmful effects of genetic inbreeding, new elk of breeding age from other populations should be introduced to increase the genetic variability the Smoky Mountain population. We propose introducing 5 new elk every ten years. An elk is sexually mature at sixteen months and can continue to mate throughout its entire lifespan of approximately 13 years[16]; therefore, a ten year interval is roughly one generation. The elk should be introduced before September; the mating season of elk, known as the rut, occurs between September and November.[15]

6.3 Reducing the Number of Accidents

By comparing the elk progress reports on the Smoky Mountains website and the accidental death data, it is evident that most, if not all, of the accidental deaths are due to road accidents. This year, the number of accidents was abnormally high and by July, three elk had already died from car accidents.[5] Two possible ways to reduce accidents are to lower the speed limit at night and to put up more Elk XING signs on the roads near where the elk live. More effective, however, is increasing driver awareness. A short driver awareness article should be included in the park brochures and posted in visitor centers. This article should include the following:

- Drive Defensively and Pay Attention!
- Most road accidents happen at dusk and dawn - animals are more active, and light levels are low. Pay more attention during these times.[2]
- Pay closer attention during calving season - late spring to early summer - since there are more inexperienced animals.[2]
- Watch the left side of the road. Studies show that drivers typically pay more attention to the right side of the road.[3]

- Elk travel in herds; if there is one elk around, chances are there are more in the area.[4]
- Signs of wildlife in the area: movement near the road, shining eyes, flickering headlights of oncoming cars or tail lights of cars in front (an animal may have just passed), roadside reflectors disappear and reappear.[3]

6.4 Preventing Birth Rate from Decreasing

Since the sensitivity analysis of our model determined that fluctuations in birth rate would exert significant change on carrying capacity, we must carefully monitor the birthrate and prevent it from decreasing. One way to do this is to increase the effective population size, which is determined by how many breeding males (N_m) and females (N_f) exist in a population and is calculated as follows:

$$Ne = \frac{4Nf\,Nm}{Nf+Nm}$$

If the essential population size remains above the minimum viable population (MVP), the population's survival is guaranteed, barring inbreeding and genetic drift. Therefore, we propose monitoring the number of male and female breeding elk, and introducing new breeding elk as necessary.[12]

Additionally, because birthing females tend to isolate themselves from the herd, we should reduce predators from areas frequented by the elk during calving season, which occurs during late spring and early summer.[15]

7. Discussion

7.1 Strengths

- Our model breaks the overall equation for population equation into five separate variables, each of which can be analyzed independently to determine its effect on population
- Our model analyzes the effect of each variable and uses the information to maximize the growth of elk population
- Our model overcompensates for the number of elk that die every year
- Our model predicts the projected elk population over 500 years in addition to determining whether the elk will survive
- To compensate for our assumptions, we tested the accuracy of our model with sensitivity analysis
- The R^2 value for every variable is feasible
- Our model corresponds to the calculated carrying capacity of elk in the GSMNP

7.2 Weaknesses

- Due to the short time span for which data is available, we made many assumptions in order to project future numbers
- Our model does not account for the effects of genetic drift
- The carrying capacity predicted by our model is sensitive to fluctuations in birth rate

7.3 Topics for future study

- Future models should create an optimization plan that consider a greater number of data points
- Accordance with the population trends of predators, prey, and the remainder of the ecosystem should be taken into consideration.
- Future studies should take into account population dispersal in the Great Smoky Mountains National Park. For example, the edge effect could affect the variables we have analyzed in order to determine elk population growth.

8. References

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Appendix

Program Old Age and Births Model (written in Python)

```
from visual import *
from visual.graph import *
from math import *

pop=140
def birth(x):
   return ((x**.5)-6.13)/(.169)
f=gcurve(color=color.cyan)

old=[25,27,0,4,2,10,8,10,13,19,19,19,25,19]

t=0
while t<175:
```

```
x=pop
pop=pop+birth(x)-old[0]
old.pop(0)
old.append(birth(x))
t=t+1
f.plot(pos=(t,pop))
print(t)
print(pop)
```