High School Mathematical Contest in Modeling 2010

Problem A: Bicycle Club

Summary

Based on existing bike stations in Chicago, Denver and Des Moines, our group aims to develop an efficient bike rental program and determine the locations in which new bike stations should be constructed.

We first determined the factors that would impact bike usage. By categorizing existing bike stations according to zip codes, we found a general positive relationship between the number of bike stations and each of the various factors we studied, such as population density, civic center popularity, and the number of tourist attractions in within the zip code. After deriving and combining the functions, we were able to arrive at a final rating system:

$$R = 3.1964e^{3.6396C} + T + (3.926285282 * 10^{-5})(1.0000709285)^{P} + \frac{0.00141}{\sqrt{0.0067125S}}$$

This rating system considers the relative weightings of each variable, and can be used to evaluate the need for new biking stations at certain locations.

To locate the different sites for upcoming bike stations that would efficiently serve the increase in bike usage and demand, we substituted data in our model to obtain ratings of the zip codes near city center. By comparing these different rankings and considering the maximum number of stations an area can carry, we found the ideal locations for the expansion of the bike program in the next five years in zip codes: 60611, 60605, 60616, 50309, 50312**,** 80204, 80205, 80206, 80209, 80210, 80211, 80218, 80223, and 80207.

Restatement Clarification of the Problem

Based on the information of the areas surrounding current B-cycle stations in Denver, Des Moines, and Chicago, we will develop a plan for an efficient bike rental program. To begin with, we will create an algorithm which includes a mathematical model that will help evaluate the need to set up new bike stations at a given location. In the process of constructing the model, we will take into consideration the existing bike paths, local attractions, other modes of transportation, bike usage, traffic, and the population density of the area surrounding each current bike station. After developing a model, we will assume that the bike usage increases at an annual rate of 30%, and then demonstrate how to implement our model to build new bike stations for the coming five years. Finally, in a short letter to the mayor, we will explain our model and analyze its strengths.

Assumptions with Rationale/Justification

1. The information provided within the zip code of the bike station provides sufficient data and accurately reflects the stations' surroundings.

   When analyzing the factors that affect the area in which each station is located, we considered the factors within the zip code area. On average, each zip code area that we analyzed had an area of 3.25 squared miles, which covers an area that is large enough such that a reasonable number of factors were considered.

2. Wealth is not a major determinant of bike usage.

   The purpose of this program is to encourage people to get around using bikes rather than cars in order to promote sustainable living. We assume that *people with high income are just as likely to use bikes as people from the lower ends of society. Thus in our model, we will not take into account the degree of wealth of the areas.*

3. Renting bikes is cheaper than paying for bus and rail fees.

   We must assume that the bike rental program is able to compete against other transportation industries. In other words, the bike industry will not be driven out of the market due to fierce competition.

4. The map we relied on is an accurate and up to date representation of the cities of study.

5. Tourist attractions are approximately equal in popularity.

   We assume that there are approximately an equal number of tourists visiting each tourist attraction. Therefore, in our model, each attraction bears an equal weight.

6. The size of each civic center is an accurate representation of its popularity.

   By estimating the number of visitors to each civic center based on the size, we may consider the area of each civic center as one of the factors that affect whether or not a bike station should be constructed at a certain location.

7. The bike stations that are listed as “Not In Use” and “Coming Soon” (on the B-cycle website) are not significant.

   We assume the bike stations are not in use because they are not making a good business due to its unideal location. Regarding the stations that are “Coming Soon”, we cannot assume that the business will be successful in that location. Thus, we will not incorporate these stations into our model.

8. Bike usage will not be affected by the seasons.

   We must assume that bike usage will grow at a constant annual rate of 30%

even during winter, when the conditions are not favorable for biking.

9. The 30% growth in usage each year is measured by bike stations:

   It is quite difficult to track the growth of the bike paths because there are no defined bike routes.

10. Other factor that may affect growth of bike stations and not taken into account in our model is neglected.

    For example, the shape of the city, the distribution of districts, and the profiles of residents in each neighborhood will not influence bike usage and its growth.

Variables and Terms Defined

- R = the rating of the suitability of a bike station at a given location
- C = the average area (per square mile) of all the civic centers in the zip code area
- T = the density (per square mile) of the tourist attractions in each zip code area
- P = the combined density of people (including the population density and the density of employees) in each zip code area
- S = the density of the student population in each zip code area
- y = number of current existing bike stations in each zip code area
- Efficiency: the ability to maximize the usage of resources
- Bike paths: the shortest distance one can take on a bike
- Bike usage: the demand for bikes as a transportation tool
- Bike Station Limit (Limit): the recommended maximum number of bike stations per zip code

Model Design and Justification

Our final model consists of two parts, a rating system (mathematical model) that generates a value that reflects how suitable a certain location is for the construction of a bike station, and an algorithm that determines the location of new bike stations. In order to develop an efficient bike rental program, we will work with the zip code areas in which each of the current bike stations (B-cycle) are located. Within each zip code, we will consider multiple factors such as the bike usage within the location, its relationship with other bike stations, and its competitiveness with other transportations.

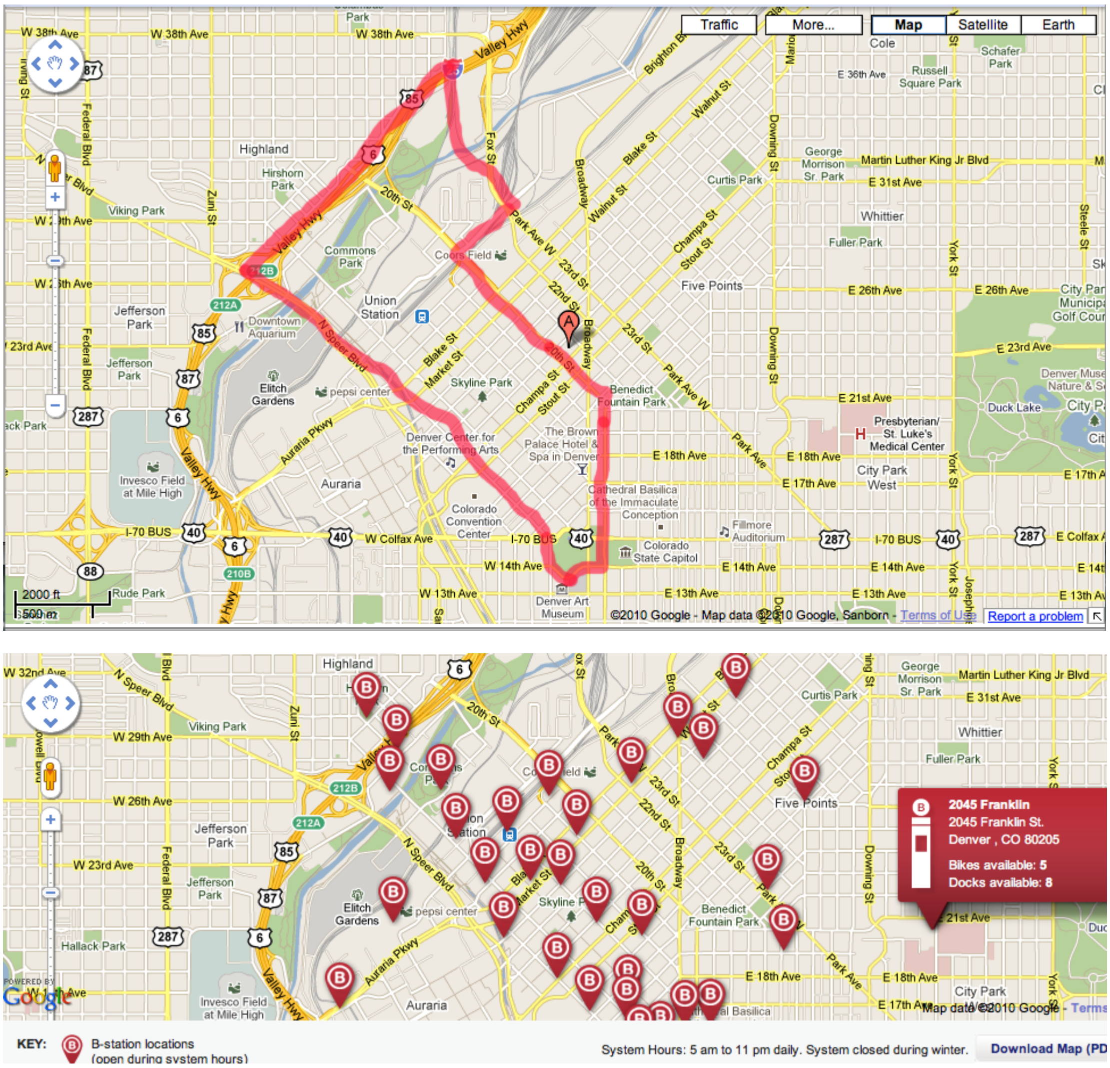
To determine the bike usage, we will focus our research on civic centers, tourist attractions, population densities (including residential and work populations), and student population densities, which are possible factors that may influence preferences for bikes. For civic centers, we will measure the sizes of shopping malls, parks, cinemas, and sports stadiums located within each zip code area. Regarding tourist attractions, we will look for museums, sightseeing locations, amusement parks, and other notable attractions. To find information about the student and working populations, we will research on schools, universities, and operating firms in each zip code area. After plotting each variable against the number of bike stations in each zip code area, we will differentiate each variable against a common variable and add these derivatives to develop a weighted rating system.

Lastly, we will demonstrate how to locate new bike stations in Chicago, Denver and Des Moines with a series of steps that determines the maximum number of stations a zip code can hold and incorporates our mathematical model.

Sorting into Zip Codes

To make our model applicable to the three different cities, we will take into account the difference in areas by sorting bike stations into zip codes as shown below. This way, we can more easily to determine the various bike stations that fall into these areas and the factors that might affect it, such as local civic centers.

Figure 1: Dividing Bike Stations into Zip Codes



Combined Density (Population Density and Employee Density)

The number and size of business firms is a factor in determining whether or not a station should be located in the area. This is because the business firms are an indication of how many employees work in the area, which affects bike usage. The website that we are obtaining the employee information from only gives a range of the number of employees and the number of establishments (in the area represented by the zip code) that employ workers in each range. To calculate the total number of employees, we will take the midpoint of the range of given values and multiply this number by the number of establishments (with the corresponding number of employees), and then find the sum of these values. We will perform these calculations for each of the zip codes in the three cities as shown below:

Zip Cod-e	1000+ (Avg: 1500)	500-999 (Avg: 749.5)	250-499 (Avg: 374.5)	100-249 (Avg: 174.5)	50-99 (Avg: 74.5)	20-49 (Avg: 34.5)	10-19 (Avg: 14.5)	5-9 (Avg: 7)	1-4 (Avg: 2.5)	Total Number of Employees
80202	4	5	16	33	66	99	94	103	362	32820
80203	0	4	3	10	11	42	59	24	170	9584
80204	2	2	5	8	8	24	18	17	35	9659
80205	0	2	3	6	7	14	21	28	45	5287
80206	1	1	3	7	16	29	25	14	33	7330
80209	1	0	1	2	4	15	14	17	80	3561

Table 1: Total Number of Employees

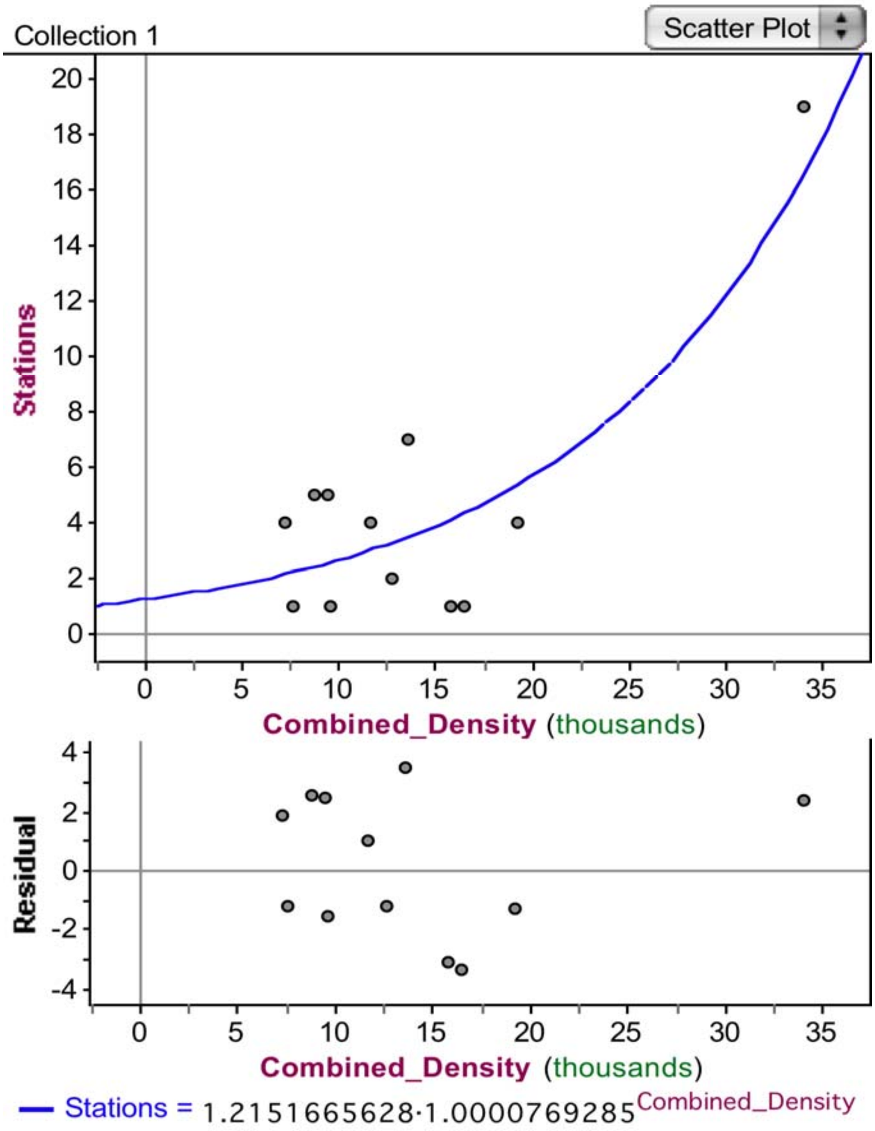
80210	2	0	1	6	11	33	25	45	86	7272
80211	0	0	1	3	6	24	19	21	30	2671
80218	3	0	1	3	6	21	21	28	47	7188
50309	2	6	12	13	14	23	19	25	94	16782
60605	4	4	1	11	11	19	19	25	72	13398
60611	4	9	10	37	47	88	82	87	187	31750
60616	3	2	4	2	8	28	34	33	98	10377

Since each zip code covers a different area of land, in order to compare the working populations, we will divide the total number of employees in each zip code by the land area of each zip code. After finding the densities of employees in each area, we will add the densities of residents in each zip code. Our results are shown below:

Table 2: Working Population Density (in each zip code)

Stations	Population Density	Land Area	Employee	Employee Density	Combined Density
19	4156	1.1	32820	29836.36364	33992.36364
4	17767	1.1	1584	1440	19207
5	6059	5.8	19659	3389.482759	9448.482759
7	7112	4.5	29287	6508.222222	13620.22222
4	8763	2.5	7330	2932	11695
1	6527	3.4	3561	1047.352941	7574.352941
5	5631	6.1	19272	3159.344262	8790.344262
1	8995	4.5	2671	593.5555556	9588.555556
1	11991	1.6	7188	4492.5	16483.5
2	6003	2	13398	6699	12702
1	13727	3.4	7077	2081.470588	15808.47059
4	1666	3	16782	5594	7260

Graph 1: Number of Stations vs. Number of Employees (in each zip code)



From the graph, it can be seen that there is an exponential relationship between the combined density (P) and the number of stations (y), modeled by the equation

$$\mathbf{y = 1.2151665628 * 1.0000769285^{P}.}$$

The residuals shown in the residual plot are randomly scattered about the x-axis, which indicates that the exponential relationship is a suitable model. In addition, the coefficient of determination, r^2= 0.79625, indicates that 79.625% of the variation in the number of bike stations can be explained by the exponential model of the number of bike stations against the combined density, indicating a relatively strong relationship.

Schools

The number of schools plays a big role in determining the location of the bike stations since schools consist of a large number of employers and students who are likely to ride bikes daily. Through the City Data website, we will find the names of the schools in each city. We will not include elementary schools because there is a low probability that elementary students would independently ride bikes to school. After obtaining a list of the schools in each city, we will search for the schools that are located in each zip code area, and then research the number of enrolled students in each school. To obtain the student population density, we will divide the total number of students (in each zip code) by the land area of each zip code. The results are shown in the tables below:

Table 3: Total Number of Students (in each zip code)

DENVER		
80202	Colorado State University	24875
	Denver School Of the Arts	902
	Metropolitan State College of Denver	24008
	TOTAL	49785
80203	Online High School	126
	Connections Academy	301
	The Art Institute of Colorado	2100
	TOTAL	2527
80204	Lake Middle School	603
	Colorado High School	178
	Emily Griffith Opportunity School	543
	Life Skills Center of Denver School	314
	West High School	967
	Ace Commnity Challenge Charter School	191
	Denver Center For International Studies School	416
	P.S.1 Charter School	380
	Community College of Denver Ccd West	13373
	La Academia at the Denver Inner City Parish	586
	Beth Jacob High School	305
	TOTAL	17856
80205	Manual High School	163
	Skyland Community High School	140

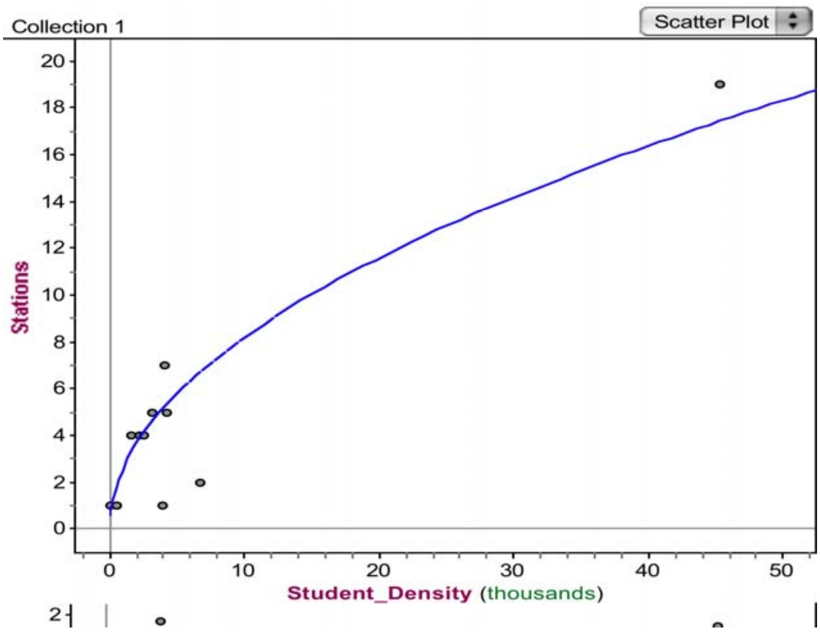
	Bruce Randolph School	680
	Community College of Denver	13373
	Wartburg College West	2461
	Crowley Foundation	459
	Denver Venture School	845
	TOTAL	18121
80206	Bromwell School	331
	Teller School	288
	East High School	2159
	The Denver Waldorf School	545
	Denver Street School	1966
	Colorado School of Traditional Chinese Medicine	901
	TOTAL	6190
80209	St. John's Lutheran School	178
	TOTAL	178
80210	Grant Middle School	461
	Merrill Middle School	639
	South High School	5407
	Iliff School of Theology	548
	University of Denver	16644
	Denver Christian Schools	202
	Mile High Academy	890
	Our Lady of Lourdes Catholic School	574
	St. Vincent de Paul School	459
	TOTAL	25824
80211	Horace Mann Middle School	210
	Skinner Middle School	357
	Contemporary Learning Academy High School	251
	Fred N Thomas Career Education Center School	371
	North High School	1080
	Emerson Street School	42
	Escuela Tlatelolco School	79
	Prep Assessment Center School	44
	TOTAL	2434
80218	Morey Middle School	712
	Academy of Urban Learning School	69
	Brooks Divinity School	75
	TOTAL	856
CHICAGO		
60605	Columbia College Chicago	10228
	East-West University	880
	Roosevelt University	2245
	TOTAL	13353
60611	Loyola University Chicago	15951
	Northwestern University	16377
	TOTAL	32328
60616	Williams Middle Prep Academy School	114
	Big Picture High - Metro School	47
	Dunbar Vocational Career Academy High School	1575
	Graham R Training Center School	186
	Perspective Charter High School	1081
	Youth Connections Charter High School	2042

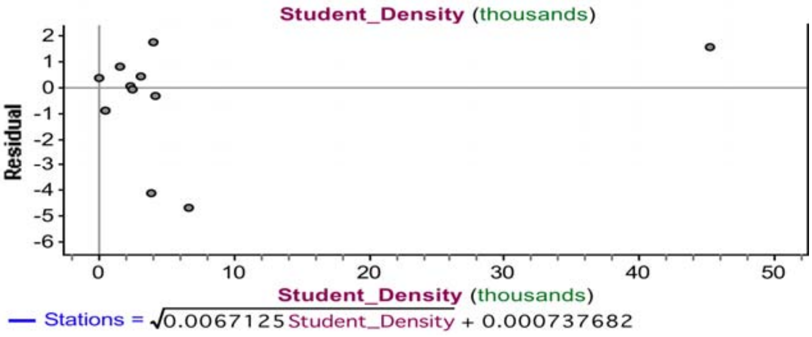
	Illinois College of Optometry	516
	Illinois Institute of Technology	7707
	Shimer College	104
	Williams Middle Prep Academy School	114
	Big Picture High - Metro School	47
	TOTAL	13372
DES MOINES		
50309	Downtown School	356
	Walnut Street School	1655
	Pace School	590
	Scavo High School	877
	Des Moines Central Campus School	1123
	TOTAL	4601

Table 4: Student Population Density

Zip Code	Stations	Land Area	Number of Students	Student Density
80202	19	1.1	49785	45259.09091
80203	4	1.1	2527	2297.272727
80204	5	5.8	17856	3078.62069
80205	7	4.5	18121	4026.888889
80206	4	2.5	6190	2476
80209	1	3.4	178	52.35294118
80210	5	6.1	25824	4233.442623
80211	1	4.5	2434	540.8888889
80218	1	1.6	856	535
60605	2	2	13353	6676.5
60616	1	3.4	13372	3932.941176
50309	4	3	4601	1533.666667

Graph 2: Number of Bike Stations vs. Student Population Density





From the graph above, it can be seen that there is a square root relationship between the student population density (S) and the number of bike stations (y), represented by the equation

$$\mathbf{y} = \sqrt{0.0067125S} + 0.000737682.$$

The r^2 value of 0.97264 indicates that 97.264% of the variations in the number of bike stations can be explained by the regression line of number of bike stations against student population density. This shows that there is a strong correlation between these two variables.

Tourist Attractions

The number of tourist attractions in each area is another factor that significantly affects bike usage. Museums, amusement parks, gardens, and scenic views attract a lot of people. Since many of the tourist attractions are similar, it is reasonable to assume that there are approximately an equal number of tourists visiting each attraction. Thus, we will count the number of tourist attractions within each zip code. In order to compare the density of the tourist attractions, we will divide the number of tourist attractions in each zip code by the land area. Our findings are shown in the table below:

Table 5: Tourist Attractions

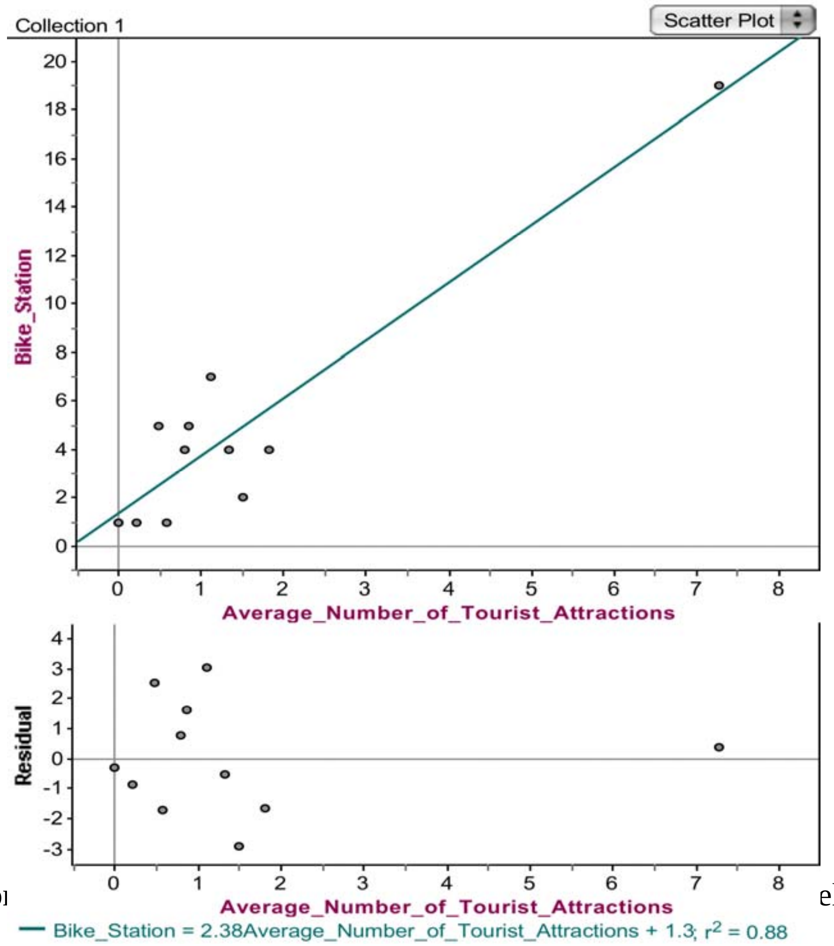
Zip Code	Tourist Attractions	Address
80202	Historic Denver Inc	1536 Wynkoop Street Suite 400A
	Museum of Contemporary Art-Denver	1275 19th Street
80203	Molly Brown House Museum	1340 Pennsylvania Street
	Vance Kirkland Museum	1311 Pearl Street
80204	Denver Art Museum	100 West 14th Avenue Parkway
	Denver Firefighters Museum Gift Store	1326 Tremont Place, Denver, CO 80204, United States
	Buckhorn Exchange	
	Museo de las Americas	861 Santa Fe Drive, Denver, CO 80204
	6 Flags Elitch Gardens	2000 Elitches Circle, Denver, CO 80204-1889
80205	Black American West Museum	3091 California Street
	Stiles African American Heritage CNTR	
	Gates Planetarium	2001 Colorado Boulevard
	Imax	
80206	Denver Museum of Miniatures Dolls & Toys	1880 Gaylord Street
	Denver Botanic Gardens	1007 York Street, Denver, CO 80206, United States
80209	None	
80210	None	
80211	Children's Museum of Denver	2121 Children's Museum Drive, Denver, CO 80211, United States
80218	None	
50309	Fort Des Moines Memorial Park	514 East Locust Street
	Historical Outfitters	511 E 6th St Suite C
	Hubbell Historical Center	507 Southwest 7th Street
	Science Center of Iowa	401 Martin Luther King Jr #
60605	Adler Planetarium & Astronomy Museum	1300 South Lake Shore Drive
	Big Time Tours & Events	828 South Wabash Avenue
	The Field Museum	1201 South Lake Shore Drive
60611	The Arts Club of Chicago	201 East Ontario Street
	Chicago Children's Museum	700 East Grand Avenue Suite 127
	Museum of Contemporary Art	220 East Chicago Avenue
60616	Chicago Sightseeing	2701 South Wabash Avenue
	Chinese-American Museum of Chicago	

Table 6: Density of Tourist Attractions (within each zip code)

Zip Code	Number of Bike Stations	Number of Tourist Attractions	Land Area	Number of Tourist Attraction Per Sq. Mile
80202	19	8	1.1	7.272727273
80203	4	2	1.1	1.818181818
80204	5	5	5.8	0.862068966
80205	7	5	4.5	1.111111111

80206	4	2	2.5	0.8
80209	1	0	3.4	0
80210	5	3	6.1	0.491803279
80211	1	1	4.5	0.222222222
80218	1	0	1.6	0
60605	2	3	2	1.5
60616	1	2	3.4	0.588235294
50309	4	4	3	1.333333333

Graph 3: Number of Bike Stations vs. Density of Tourist Attractions



Fro[illegible]elationship between the number of tourist attractions (T) and the number of bike stations (y), represented by the equation

$$y = 2.3808T + 1.3257.$$

The r^2 value of 0.87511 indicates that 87.511% of the variations in the number of bike stations can be explained by the regression line of number of bike stations against number of tourist attractions. Thus, there is a relatively strong correlation between these two variables.

Civic Centers

Civic centers not only attract tourists, but also attract locals. The entertainment centers that we will consider include cinemas, shopping malls, parks, plazas, stadiums, golf courses, and libraries. Since different-sized civic centers attract a different number of people, we will assume that the area of the civic center is a good indication of the number of people visiting each center. Thus, we will find the total area of the civic centers in each zip code. Similar to the calculations we performed for working population, student population, and tourist attractions, we will divide the total area of civic centers in each zip code by the land area of each zip code to obtain the density of these civic centers. Our results are presented in the table below:

Table 7: Civic Centers

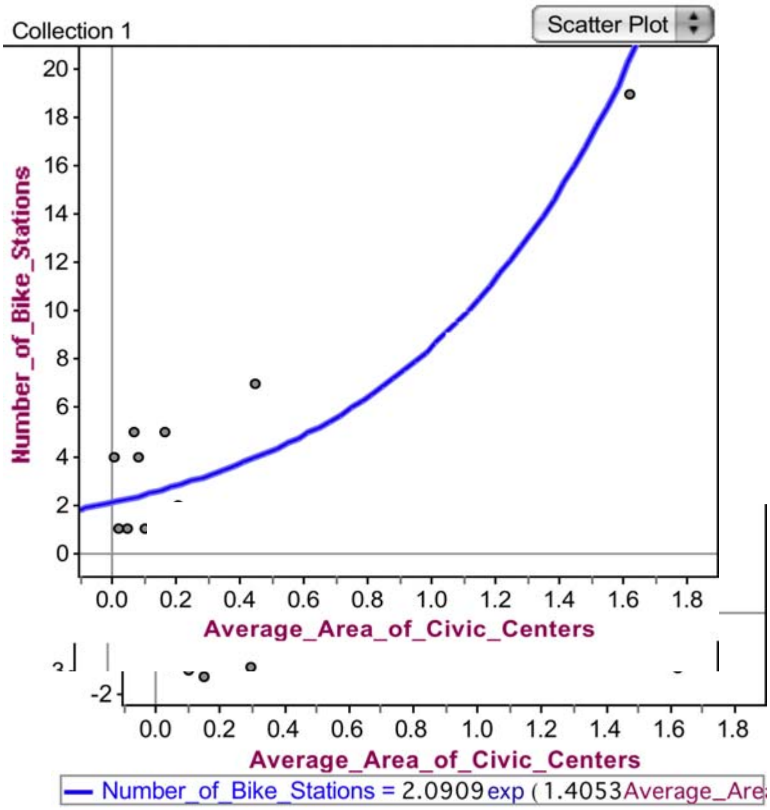
Zip Codes	Civic Centers	Area
60605	Butler Field, Buckingham Fountain, Logan Monument, Grant Park	1.01
	Dearborn Park	0.0196
	Park 479	0.00444
	Roosevelt Park	0.00711
	Merle Reskin Theatre, Chicago, IL, United States	
60611	Waveland Avenue Golf Course	0.88
	Lincoln Park	0.39
	Cityfront Plaza	0.028
60616	Woodland Park	0.00444
	Dunbar Park	0.0857
	Douglas Monument Park	0.00741
	Prairie Avenue Historic District	
	Armour Square Park	0.04
	Lake Meadows Park	0.0264
	Groveland Park	0.00667
80202	United Nations Park	0.106
80203	Governors park	0.00612
80204	Lincoln Park	0.0765
	Sunken Gardens	0.07
	Barnum Park	0.334
80205	City Park	; 1.3km2
	Saint Charles Place Park	0.000612
	Fuller Park	0.0306
	Schafer Park	0.0329

Curtis Park	0.0367

Table 8: Total Area of Civic Centers (in each zip code)

Zip Code	Total Area of Civic Centers	Number of Bike Stations	Land Area	Average Area of Civic Centers
80202	1.78186	19	1.1	1.619872727
80203	0.00612	4	1.1	0.005563636
80204	0.9542	5	5.8	0.164517241
80205	2.00812	7	4.5	0.446248889
80206	0.212	4	2.5	0.0848
80209	0.34745	1	3.4	0.102191176
80210	0.4024	5	6.1	0.065967213
80211	0.1073	1	4.5	0.023844444
80218	0.236305	1	1.6	0.147690625
60605	0.4115	2	2	0.20575
60616	0.17062	1	3.4	0.050182353
50309	0.0243	4	3	0.0081

Graph 4: Number of Bike Stations vs. Density of Civic Centers



Evidently, there is an exponential relationship between the average area of civic centers (C) and the number of bike stations (y), modeled by the equation y = $2.0909e^{1.4053C}$. In addition, there are no obvious patterns shown by the residuals in the residual plot, which indicates that the exponential model is a suitable fit. Although the coefficient of determination of r^2=0.4449 does not indicate a very strong fit, from the graph, it can be seen that the data points generally follow a exponential pattern.

Calculations

In order to develop a rating system that includes all of the factors that we considered, we will differentiate each of the models obtained above, and then add them up to determine the final rating. The derivative of each function represents the change of the number of bike stations in proportion to the change in each variable, which reflects the weight of the variable in the rating system. After determining the derivative of each function, we will compare each of the derivatives with one set variable. In our model, we will choose our "constant" variable to be the average number of tourist attractions due to its simple derivative. In order to compare all the variables with each other, we will find the ratio between the derivative of the variable and that of the average number of tourist attractions. For instance, to determine the extra weight of the average area of civic centers, we will divide $\frac{dy}{dC}$ by $\frac{dy}{dT}$:

$$\frac{dy}{dC} \times \frac{dT}{dy} = \frac{dT}{dC}$$

Hence, we would achieve a mathematical representation of how much more influential a square mile of civic center is than an increase in the average of tourist attractions is to the number of bike stations. We will repeat the same method for the remaining two variables.

First, we will calculate the derivative of the number of bike stations against density of tourist attractions:

$$y = 2.3808T + 1.3257$$

$$\frac{dy}{dT} = 2.3808$$

Next, we will calculate the derivative of the number of bike stations against combined density:

$$y = 1.2151665625*1.0000769285^{P}$$

$$\frac{dy}{dP} = (9.3477*10^{-5})(1.0000709285)^{P}$$

$$\frac{dy}{dP} \div \frac{dy}{dT} = \frac{(9.3477*10^{-5})(1.0000769285)^{P}}{2.3808}$$

$$\frac{dT}{dP} = (3.926285282*10^{-5})(1.0000709285)^{P}$$

Similarly, we will calculate the derivative of the number of bike stations against student population density:

$$y = \sqrt{0.0067125S} + 0.000737682$$

$$\frac{dy}{dS} = \frac{0.00335625}{\sqrt{0.0067125S}}$$

$$\frac{dy}{dS} \div \frac{dy}{dT} = \frac{0.00335625}{\sqrt{0.0067125S}} * \frac{1}{2.3808}$$

$$\frac{dT}{dS} = \frac{0.00141}{\sqrt{0.0067125S}}$$

Finally, we will calculate the derivative of the number of bike stations against average area of civic centers:

$$y = 2.0909e^{1.4053C}$$

$$\frac{dy}{dC} = 7.610e^{3.6396C}$$

$$\frac{dy}{dC} \div \frac{dy}{dT} = \frac{7.610e^{3.6396C}}{2.3808}$$

$$\frac{dT}{dC} = 3.1964\, e^{3.6396C}$$

Combining these functions brings us to our final model:

$$\mathbf{R = 3.1964}e^{\mathbf{3.6396C}}\mathbf{ + T + (3.926285282 * 10^{-5})(1.0000709285)^{P} + }\frac{0.00141}{\sqrt{0.0067125S}}$$

Determining Where to Add New Locations in Five Years

To satisfy all the prerequisites, using our model, we will develop an algorithm that helps determine where to locate new bike stations for the next five years.

1. How many new stations do we need to add each year?

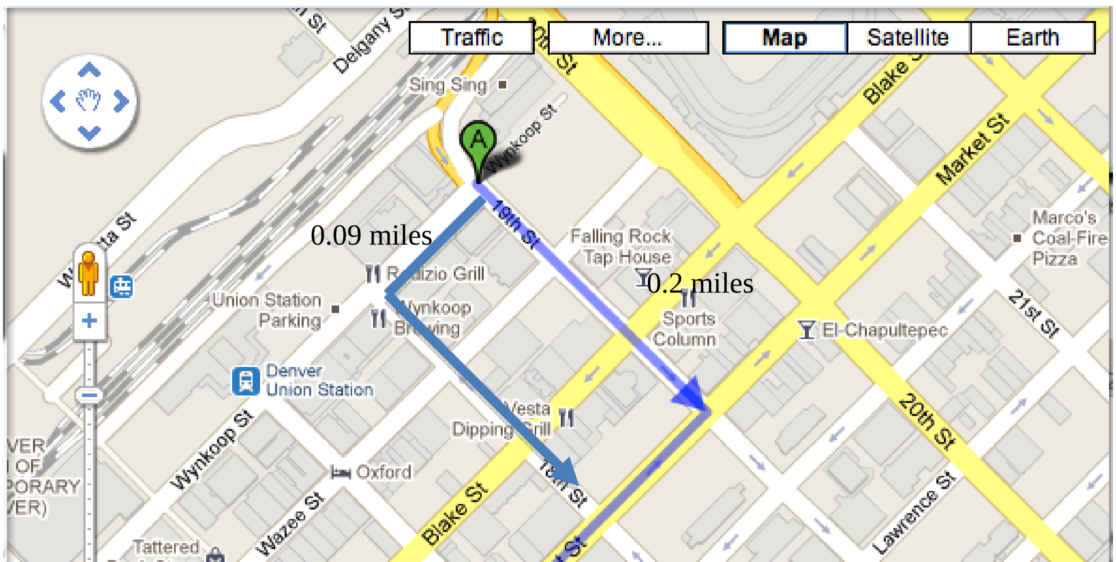
With an assumed 30% growth in bike stations each year, we can calculate the number of new bike stations by multiplying the original number in the city by 1.3.

2. Do we need to add more bike stations to existing zip codes?

When deciding whether or not we want to either expand existing bike stations or find new bike stations, we must take the existing bike paths and stations into account.

Before we answer the above question our first step is to determine at what distance from the old locations the new stations should be located. From our knowledge of the existing stations, we will estimate the walking distance between the closest stations.

Figure 2: Determining distances



0.2 miles

0.3 miles

To be more efficient, we will only consider a random sample of 20 to represent the 54 stations. We will label each station from 1- 54 and use a random digit table to choose 20 stations. From those 20 stations, we will estimate the shortest walking distance to the closest station by drawing tree diagrams that are weighted by distance as shown below. After generalizing our results, we will obtain a good idea of where we should locate our new stations. Below is a table that shows the approximated walking distances from the random stations:

Table 9: Walking Distances from Random Stations

Random Stations	Walking distances
1	0.9
2	0.3
3	0.6
4	0.8
5	0.2
6	0.4
7	0.6
8	0.7
9	0.2
10	0.3
11	0.2
12	0.5
13	0.9
14	0.8
15	0.3
16	0.9
17	0.1
18	0.1

19	0.3
20	0.7
Average	0.49

As seen from above, the average shortest distance of a bike station to another bike station is 0.49 miles. This makes sense because it is not favorable to have a bike station that is too far away. This may make bikers feel obligated to travel a long way to the next bike station in order to store their bikes. Therefore, the new bike locations should be roughly 0.49 miles away from old bike locations.

Since our model is based on zip codes, we must also consider the questions of how many bike stations we need per zip code and whether or not we need to add any more bike stations to existing zip codes.

Given that the average distance between stations is 0.49 miles, we can conclude that the approximate area around a bike station is:

$$0.49^2 = 0.2401 \text{ miles}^2$$

Therefore, the number of stations per zip code would equal:

$$NumberOfBikeStations = \frac{AreaOfZipCode}{0.2401}$$

After we find out the set number of bike stations, we will compare them with the number of existing bike stations and determine where the new stations should be added.

a) If the existing number of bike stations is less than number of bike stations we find, new bike stations should be located in that zip code.

b) If the existing number of bike stations is equal to or greater than the number of bike stations we find, then we should stop adding new bike stations in that zip code and proceed to the next step.

3. Adding new bike stations to zip codes

Using the following model, we will calculate the ratings to determine the need for bike stations in each zip code:

$$\mathbf{R = 3.1964}e^{\mathbf{3.6396C}} \mathbf{+ T + (3.926285282 * 10^{-5})(1.0000709285)^{P} +} \frac{0.00141}{\sqrt{0.0067125S}}$$

We will place the new bike stations in the zip code with the greatest rating and when the limit is reached, we will proceed to add new bike stations to the area with the second greatest rating.

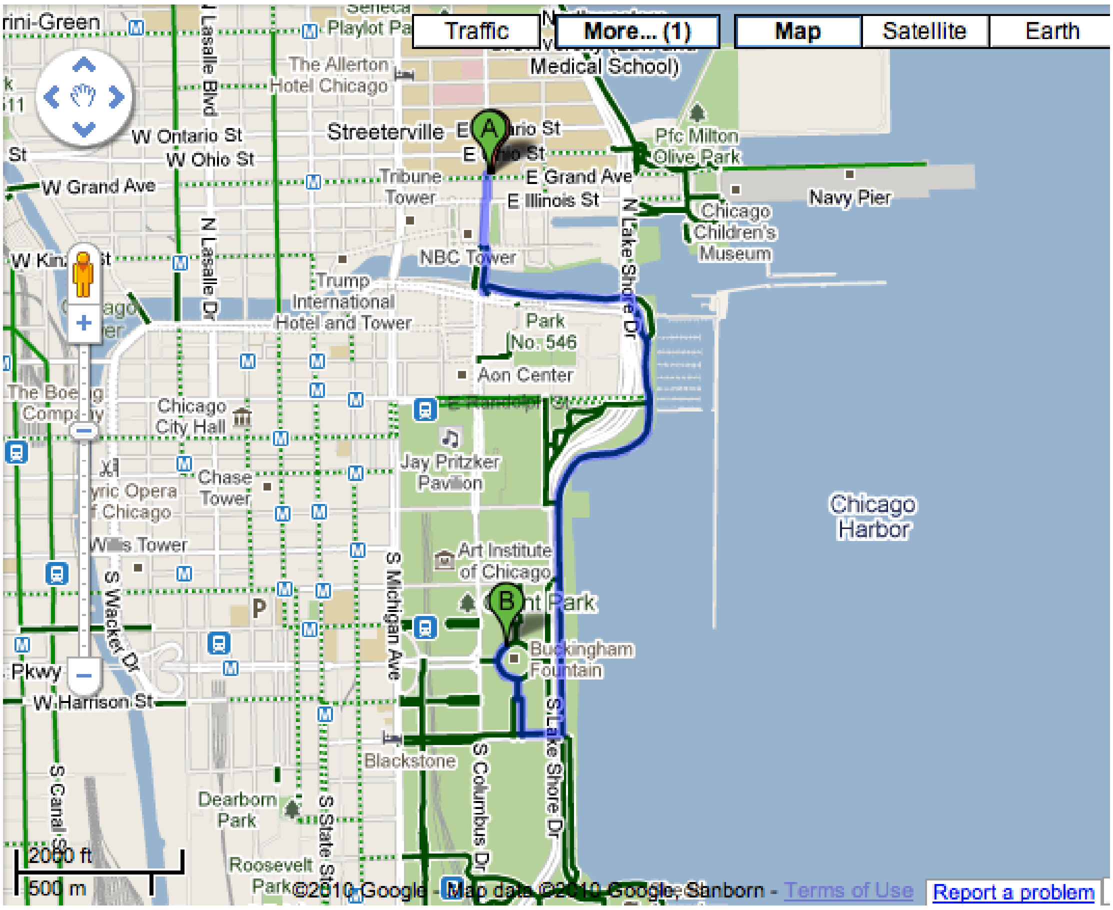
4. Where to place the new bike stations within the zip codes.

When we plan new bike locations, we must consider how to make the stations more competitive in the area so that people would choose bikes over buses and other forms of transportation. One reason that people would choose bikes over other means of transportation is time. Often times, not only are bus or transit routes located in inconvenient places, requiring people to walk to the stations, they also entail long waiting times and congested traffic. Therefore, bike stations should aim to be located at places that are most convenient and minimize travelling time. This is evident when we look at the existing bike stations. For example, when we compare the shortest bike time, walk time and bus/ transit time, even though bike distances may be longer, the bike time is always less than that of other travelling means.

Table 10: Traveling Times for Different Modes of Transportation

Bike Distance	Bike Time	Walk Distance	Walk Time	Bus/Transit Time
0.9	5	0.9	18	18
0.7	6	0.6	12	8
0.7	6	0.7	15	9
0.6	2	0.3	5	5
0.9	17	0.9	6	20
0.9	5	0.9	19	18
0.7	3	0.7	12	4

Figure 3: Examining bike routes



New Locations in Chicago

1. How many new stations do we need each year?

Table 11: New Stations in Chicago

Year	Total stations	Total Rounded	New stations each year
0	5		
1	6.5	6	1
2	8.45	8	2
3	10.985	10	2

4	14.2805	14	4
5	18.56465	18	4

2. & 3. Adding New Bike Stations

Table 12: Ratings of Zip Codes in Chicago

Zip Code	Land Area	Population Density	Limit	Rounded Limit	Current Number	Rating
60605	2	6003	8.329862557	8	2	5.768
60611	1.6	16739	6.663890046	6	2	12.116
60616	3.4	13727	14.16076635	14	1	4.018

The first station that will be needed in the following year due to the thirty percent increase in bike usage should be added into the zip code area of 606011. This is because the rating of 60611 is the highest amongst the zip codes while the current number of bike stations in that area has not reached the limit we calculated. During the second year, two new stations will also be added in the same zip code area because the need for bike stations in that area has not been fully satisfied. In the third year, one new bike station should be set in 60611 and another in 60605. This is because after adding the new bike station in 60611, the limit will be reached and the zip code area with the next highest rating should then be taken into account. This method is then repeated for five years and by the end of the fifth year, four new stations should be added to 60611, six stations to 60605, and three stations to 60616.

New Locations in Des Moines

1. How many new stations do we need each year?

Table 13: New Stations in Des Moines

Year	Total stations	Total Rounded	New stations each year
0	4		
1	5.2	5	1
2	6.76	6	1
3	8.788	8	2
4	11.4244	11	3
5	14.85172	14	3

2. & 3. Adding New Bike Stations

Since there is currently only one zip code area in Des Moines in which the firm has implemented its bike program, we need to consider whether the new stations constructed in the next five years will surpass the limits of that zip code. In total, after the next five years, ten new stations will be added to serve the increase in bike usage and the demand in the local market. However, there are only eight spaces remaining in the current zip code area. Thus, new areas must be considered in order to decide the locations of the remaining two new stations.

Table 14: Ratings of Zip Codes in Des Moines

Zip Code	Land Area	Limit of Bike Stations	Rounded Limit	Current Number	Rating
50309	3	1666	12	4	4.566829645
50310	7.6	31.65347772	31	0	3.528324908
50311	3.7	15.41024573	15	0	4.569023901
50312	5.8	24.15660142	24	0	5.123065457
50313	18.3	76.2182424	76	0	3.556095461
50314	2.5	10.4123282	10	0	3.78654546
50315	9.4	39.15035402	39	0	3.678546345
50316	3.6	14.9937526	14	0	4.349831982
50321	12	49.97917534	49	0	2.94543956

As seen from the table above, the remaining two new stations should be added to zip code area 50312, which has the highest rating among the surrounding areas. Therefore, in the city of Des Moines, the bike program will be expanding in two zip code areas, 50309 and 50312, in the next five years.

New Locations in Denver

1. How many new stations do we need each year?

Table 15: New Stations in Denver

Year	Total stations	Total Rounded	New stations each year
0	47		

1	61.1	61	14
2	79.43	79	18
3	103.259	103	24
4	134.2367	134	31
5	174.50771	174	40

2. & 3. Adding New Bike Stations

Table 16: Ratings of Zip Codes in Denver (Area with Existing Bike Stations)

Zip Code	Land Area	Limit	Rounded Limit	Current Number of Stations	Combined Density
80202	1.1	4.581424406	4	19	33992.36364
80203	1.1	4.581424406	4	4	19207
80204	5.8	24.15660142	24	5	9448.482759
80205	4.5	18.74219075	18	7	13620.22222
80206	2.5	10.4123282	10	4	11695
80209	3.4	14.16076635	14	1	7574.352941
80210	6.1	25.4060808	25	5	8790.344262
80211	4.5	18.74219075	18	1	9588.555556
80218	1.6	6.663890046	6	1	16483.5

Zip Code	Average Area of Civic Centers	Number of Tourist Attraction Per Sq. Mile	Student Density	Rating
80202	0.625436357	7.272727273	45259.09091	38.40920325
80203	0.002148132	1.818181818	2297.272727	5.040182705
80204	0.063520462	0.862068966	3078.62069	4.890231428
80205	0.172297659	1.111111111	4026.888889	7.095667968
80206	0.032741463	0.8	2476	4.401361757
80209	0.039456234	0	52.35294118	3.692459137
80210	0.025470083	0.491803279	4233.442623	3.999018703
80211	0.009206391	0.222222222	540.8888889	3.528357961
80218	0.057023669	0	535	3.934523498

Though zip code area 80202 has an outstandingly high rating, its current number of stations has surpassed the approximate limit of bike stations that neighborhood should have. Thus, for the following year, new bike stations should be added to other areas that have not reached their limit. The next highest rating falls on area 80205. Eleven stations can be constructed in 80205 during the first year and three other stations should be added to area 80204. Then, the next 18 new stations in the second year can be distributed throughout areas 80204 and 80206. In the third year, 24 new stations can be established in 80206 and 80210. Thirty-one stations can be added to 80218, 80209, and 80211 in the fourth year. Lastly, during the fifth year, 80211, the zip code area with the lowest rating in the above table, will reach its limit after four of the 40 new stations are added to its neighborhood. As a result, the company would need to expand to other nearby zip code areas. The following table shows our consideration of the surrounding areas to determine where the remaining 36 stations should be located.

Table 17: Ratings of Zip Codes in Denver (Possible Areas of Expansion)

Zip Code	Land Area	Limit	Rounded Limit	Current Number of Stations	Rating
80216	14.9	61.82572614	61	0	3.163498795
80207	4.3	17.84232365	17	0	4.887284723
80220	8.2	34.02489627	34	0	3.094730395
80246	1.7	7.053941909	7	0	2.99834851
80223	5	20.74688797	20	0	5.13723897
80219	7.5	31.12033195	31	0	3.78380513
80214	2.4	9.958506224	9	0	3.000212333

As the table shows, zip code area 80223 has the highest rating among the surrounding areas. Its limit is 20 and this would be also fulfilled during the fifth year. The remaining 16 new stations will then fall into the second highest rating area – 80207. There are 17 spaces in this zip code area; thus, this area will be sufficient for the construction of these new sites. In sum, within the next five years,19 new stations should be constructed in 80204, 11 stations in 80205, 6 stations in 80206, 13 stations in 80209, 20 stations in 80210, 17 stations in 80211, 5 stations in 80218, 20 stations in 80223, 16 stations in 80207.

All the new stations in the next Five Years

Chicago

- 4 stations in 60611
- 6 stations in 60605
- 3 stations in 60616.

Des Moines

- 8 stations in 50309
- 2 stations in 50312

Denver

- 19 new stations should be constructed in 80204
- 11 stations in 80205
- 6 stations in 80206
- 13 stations in 80209
- 20 stations in 80210
- 17 stations in 80211
- 5 stations in 80218
- 20 stations in 80223
- 16 stations in 80207

Model Testing and Sensitivity Analysis

To test our mathematical model, we will make sure that each of our relationships makes logical sense.

First of all, according to our model, there is an increasing exponential relationship between the combined density (sum of employee density and population density) (P) and the number of bike stations (y). This makes sense because as the number of people in an area increase, bike usage is higher thus there must be more bike stations to accommodate everybody’s needs.

Regarding schools, there is a square root relationship between the student population density (S) and the number of bike stations (y). This relationship makes sense because students often ride bikes to school thus a greater student population density would result in higher bike usage. As a result, there would be a greater number of bike stations. The rationale behind the square-root relationship is because as the population gets more and more dense, the number of bike stations continues to increase but at a decreasing rate due to the limitations in resources, land, and cost.

On the other hand, there is a positive linear relationship between the number of tourist attractions (T) and the number of bike stations (y). The rationale behind is similar to those above. Tourists may choose to bike so that they can enjoy the view along the way. Therefore, as the number of tourist attractions increase, the number of tourists increase, causing bike usage to increase.

Finally, there is an exponential relationship between the average area of civic centers (C) and the number of bike stations (y). Since parks are a huge part of recreational life and are great venues for biking, the greater the average area of civic centers, the greater the number of bike stations. The exponential relationship may also be explained by the civic center's popularity. As we assumed, the larger the civic center, the more popular it is. The more people there are, the more bike stations available.

Our mathematical model makes sense because by taking the derivative of the model of each subcategory, we are finding the rate of change. This means that the steeper the slope, the greater the value (or weight) of the subcategory thus the bigger role it will play in determining the final rate. Also, the results (rating for each zip code) make sense. The rating is higher for zip codes with more stations. For example, 80202 in Denver has a high rating of 38.4. It also has an exceptionally large number of biking stations. The high rating indicates the great needs of the area, hence the large number of biking stations.

Strengths and Weaknesses

One of the greatest strengths of our model is that we included a lot of factors. We took into account resident population density, employee population density, student population density, tourist attractions, other modes of transportation, and civic centers. We conducted in depth research of the information of each area. We considered even the small industries and schools, and included a wide variety of tourist attractions and civic centers into our model. This greatly contributes to the accuracy of our model.

Another strength of our model is that we weighted each of the variables after examining the relationships between each of the variables with the number of bike stations. Weighting the variables increases the accuracy of our final rating system.

By providing a mathematical model to evaluate the need of a new bike station at a certain location, our approach will not only help answer where, but will also demonstrate why to locate the new bike stations. Additionally, when locating the new stations, we took the existing B-cycle model into consideration. For example, we analyzed the average distances and took bus routes and other transportation into consideration by analyzing the time required to travel from one station to another. Our model considers the fact that areas with existing bike stations still need additional stations in order to satisfy the demand for bikes, which accurately models the real-world situation.

The main weakness of our model is that we did not include enough data points. Since we only worked with 12 zip codes, we only have 12 data points to determine each relationship. To strengthen our model, we could have researched different bicycle programs in more cities in order to obtain more information about bike stations in a wider variety of locations. This would not only provide us with more data points to work with, but also ensure that our model would be more generalized thus be more applicable to other cities.

Another weakness of our model is that we only considered the zip code in which the bike station is located. This may not be a truly accurate representation since there may be some factors lying outside of the zip code area that affect bike usage within the area.

Letter to the Mayor

Dear Mr. Mayor,

We are a group of enthusiastic high school math students who are aware that the selfish actions of humans are exacerbating the state of our environment and putting our Earth at stake. In order to promote a sustainable future, we would like to propose an efficient bike rental program that would encourage people to travel around by bike rather than by car.

By studying the areas where current bike stations are located, we have developed a mathematical model that could help determine whether or not a bike station should be constructed at a certain location. The following model gives the rating of how well a bike station would suit a certain area,

$$R = 3.1964e^{3.6396C} + T + (3.926285282 * 10^{-5})(1.0000709285)^{P} + \frac{0.00141}{\sqrt{0.0067125S}}$$

where R = the rating of the suitability of a bike station,

C = the average area (per square mile) of all the civic centers in the zip code,

T = the density (per square mile) of the tourist attractions

P = the combined density of people (including the population density and the density of employees)

and S = the density of the student population

The strength of our model is that it takes a lot of factors into account. By conducting in depth research of the information of each area, we incorporated even the smallest industries, schools and tourist attractions into our model. In addition, our final model does not just simply consist of the sum of the individual ratings of each sub-category, it also considers the weighting of the individual ratings based on the strength of the relationship between each category and the number of existing bike stations.

We also propose an algorithm in conjunction with our model to help locate new bike stations in the next five years. This algorithm considers the existing bike stations by comparing the maximum number of bike stations per zip code using

$$NumberOfBikeStations = \frac{AreaOfZipCode}{0.2401}$$

and the existing numbers of bike stations. It also adjusts according to current transportations in the area. Therefore, our proposal not only accounts for the expand of the program, but can also satisfy the needs within each zip code appropriately, given current real world situations.

Mr. Mayor, I strongly encourage you to adopt this efficient bike program not only to reduce traffic, but also to save our Earth.

Sincerely,

High School Mathematical Modeling Team 2543

13th November, 2010

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